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TRUSS STRUCTURE OPTIMIZATION
USING
REAL-CODED GENETIC ALGORITHM

by

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TRUSS STRUCTURE OPTIMIZATION USING REAL-CODED GENETIC ALGORITHM

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CERTIFICATE

It is certified that the work contained in the thesis entitled, "**TRUSS STRUCTURE OPTIMIZATION USING REAL-CODED GENETIC ALGORITHM**", has been carried out by **SURENDRA GULATI**, under my supervision and this work has not been submitted elsewhere for a degree.

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Abstract

Genetic Algorithm (GA) is a stochastic search method based on principles of natural selection and Darwin's principle of survival of the fittest. GA does not require gradient information which makes it easier to use. GA is a population based method, that is, it processes various designs at every stage, due to which it is less susceptible to pitfalls of convergence to a local minima.

In the present work the real coded GA has been applied for optimizing the cross-sectional size, topology, and configuration of 2D and 3D trusses, using a novel ground structure approach. For all problems the objective has been kept as minimization of the weight of a truss. For the problem of optimizing the topology and sizing of members for minimum weight truss, continuous design variables are used to denote both presence or absence, and area of any member in a ground structure. For the configuration optimization nodal locations are also treated as continuous variables. Stress and displacement constraints are applied in all problems using a penalty based transformation of function. FEM analysis is used to calculate nodal displacement and member stresses for different loading conditions.

Standard 2D and 3D test problems are considered from the literature and obtained results are compared for different population sizes in the present problem and also compared with those reported in the literature. Based on these results the proposed methodology is found to be satisfactory for optimizing complex structural problems.

Acknowledgements

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Dr. Kalyanmoy Deb introduced me to Genetic Algorithm and other traditional methods of optimization. He provided me the C code of real genetic algorithm which I have used in the present work. I again sincerely express my gratitude to him.

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Surendra Gulati

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Chapter 1

Introduction

1.1 General

Over the years, numerous optimization techniques have been proposed for improving the design of structural systems. Due to the need to handling a wide spectrum of problems of sizing, shape and topology optimization of skeletal structures, the field still continues to be an active area of research.

Structural optimization techniques are evolved over three decades of research. Initially the work was limited almost exclusively to structures of specified geometric configuration. A notable exception was presented by Dorn, Gomory and Greenberg in 1964 . These researchers begin by choosing a set of admissible joint locations in 2D space and then determining how these structures must be connected. The design problem was formulated as a linear programming problem. Dobbs and Felton in 1969 extended the work through the use of non-linear programming methods. Pederson in 1969 presented a method for shape optimization in which joint coordinates were treated as independent design variables.

Recently, genetic algorithms (GAs) have been used for solving a variety of structural design problems. They include the optimum design of 10-bar truss by Goldberg and Samtani (1986) , truss-beam roof structure by Jenkins (1991), and welded beams by Deb (1991) . The application of GA to solve

relatively large problems, has been explored by several researchers. Rajeev and Krishnamoorthy (1992) used GAs to find minimum weight truss systems with discrete design variables having stress constraints only. Hajela, Lee, and Lin, (1993) solved the problem in two stages of optimization using GA.

1.2 Present Work

Present work extends the prior works in an attempt to enhance the applicability of GAs to design practical skeletal structures. To make the GA solutions attractive as a design methodology (as an alternative to traditional design optimization methodologies) would require that sizing shape and topology aspects of structural optimization be addressed simultaneously. In 1995, Chaturvedi, under the guidance of K. Deb and S. Chakrabarti, has implemented a combined algorithm using GAs and solved a couple of simple truss-structure problems. In most of the present study, that study is extended and applied to different types of larger truss-structure optimization problems. Particular attention is paid to sizing and topology and in some cases all three optimizations are carried out simultaneously. In sizing and topology optimization structures with many possible members and predetermined possible location of joints is used to determine optimal subset of connectivity and optimal member areas. These structures are called as ground strustures. Continuous design variables are used to denote presence or absence of member and and area of that member, if present, simultaneously. Later, shape optimization is also considered with sizing and topology optimization, using separate continuous design variables to denote the change of spatial position of nodes.

Real-coded GA (Deb and Agrawal, 1995) is used with continuous or discrete probability distribution in different cases. Discrete probability distribution was used in some cases to generate discrete areas of members, hence increasing the applicability of algorithm to the areas where material is available in standard sizes.

Since the GA is best suited to unconstrained maximization of objective function, it is necessary to impose the transformations to apply constraints, and minimization of weight. For taking care of constraints a penalty based transformation is used and for minimization, a suitable fitness function is assumed.

A number of test problems from literature are solved using the proposed GA and results are compared with previous results and the aspect of computational effort is also discussed.

Chapter 2

Truss Structure Optimization

Optimum design of truss structure is basically the problem of optimizing the set of design variables, describing the properties of truss, that together optimize the stated objective function. Usually the objective function would be cost-based, but this is often difficult to achieve since realistic cost information are not always available. For this and other reasons, workers in this field have usually adopted other criteria to compare designs. Often, weight of the material used in a truss forms the objective.

In general, optimal structure design aims at arriving at designs such that the weight of a truss is minimum. The three optimization procedures are followed, either simultaneously or separately:

Size optimization : Cross-sectional areas of members are used as design variables.

Configuration optimization : Coordinates of joints are used as design variables.

Topology optimization : Connectivities of joints and member cross-sectional area are design variables.

2.1 Methodologies Used for Structural Optimization

Truss optimization can be broadly classified into two categories, first is that which results in grid-like continua, and second is the determination of optimal element connectivity from a number of finite, albeit large number of possible connectivities. The later approach, which is used in present work, is described as a **ground structure approach** in the literature.

Ground Structure contains many possible locations of node points and members, from which an optimal structure must be derived. Since all the members in optimized truss must be there in ground structure, so ground structure may be defined as discrete version of structural universe. Different approaches are adopted by different workers to find this optimal subset of ground structure. Some of these methods are discussed in following few paragraphs.

A review of literature on optimal design of skeletal structures (Toppings, 1983) shows that a major amount of work, carried out in early the stage of structural optimization, is in the area of size optimization, in which member sizes (cross-sectional properties) are allowed to vary with constant configuration and topology. Whereas, in field of configuration and topology optimization limited amount of work has been done. This is due to the difficulty in adopting mathematical programming techniques to handle different types of design variables, corresponding to sizing, topology and configuration optimization.

Simultaneous optimization of topology and sizing of members first presented by Dorn, Gomory, and Greenberg (1964). They defined the ground structure by connecting all nodal points with members. Member forces were assumed as variables. The problem reduced to a linear programming (LP) problem. Number of variables in this problem was equal to twice the number of members. The constraints were constructed from the equation of equilibrium at all joints of the structure. The members areas were obtained by dividing optimal member force by maximum feasible stress.

Method proposed by these authors gives the statically determinate and

fully stressed structure, which means that displacement constraints can not be considered in rational way. Other disadvantage is that this method is limited to only single loading condition.

Dobbs, and Felton (1969) extended the method. to solve statically indeterminate structures subjected to multiple loading conditions. Member cross-sectional areas were used as design variables. Problem became nonlinear programming problem (NLP), with linear objective function but nonlinear constraints.

2.1.1 Structural Design via Genetic Algorithm

Genetic algorithm is a stochastic direct search method based on principles of natural selection and survival of the fittest. It generates set of design parameters and modify them according to their fitness. One of the main problem in structural design is related to the handling of constraints like related to stress, deflection, dimensional relationship and other variables. In GA the constraints are conventionally handled by 'penalty' approach, where constraint violating set of variables (string) are penalized to deter further use of that string (survival of fittest). One other advantage in using GA is easier handling of multiple loading condition. Multiple loading condition can be taken care by considering most critical loading for purpose of assigning fitness to any particular set of variables.

Jenkins (1991) applied the GA to find minimum weight design of trussed beam structure. He used binary GA to optimize the problem of 6 discrete variables, related to sizing of member (cross-sectional properties) and configuration (spatial position of members). Based on results author concluded that GA gives much improved approach to optimal design. Rajeev and krishnamoorthy (1992) applied the genetic algorithm for getting size optimal 2D and 3D structures using discrete design variables.

Hajela, Lee, and Lin (1993) implemented the GA to optimize the sizing and topology of trusses using two level approach. In first level of problem only kinematic constraints were used to generate kinematically stable topologies.

Binary GA was used with string length equal to number of possible members in ground structure. Presence or absence of any member denoted by 1 or 0 value at corresponding bit of string. In second level of implementation cross-sectional areas also included as design variables, and coded to binary string of required length according to precision required. Area optimization in second level was carried out for all the topologies generated in first level. Stress and displacement constraints were imposed in second level only.

Rajan (1995) developed the procedure for combined sizing, shape and topology design of space trusses using GA. Author used the discrete and continuous design variables to denote the cross-sectional areas of members in context of size optimization. For optimization of configuration, nodal location of ground structure were treated as separate continuous design variables. To meet the requirement of topology optimization Boolean design variables were used to find optimal connectivity. Transformation methods using exterior penalty were used to handle constraints.

2.2 Present Approach

In the present work, all three aspects in optimal design of trusses are taken care of simultaneously. These aspects are sizing, shape and topology optimal design. Both stress and displacement constraints have been taken care by applying exterior penalty transformation, as described in next chapter. Real coded genetic algorithm is used with either continuous or discrete probability distribution in all problems. All present problems are solved by using ground structure approach as discussed later.

Concept of basic and non-basic nodes is adopted from the work of Chaturvedi, (1995) to save the computational cost. Here the basic nodes are defined as nodes of ground structure having either concentrated load or support. Presence of all basic nodes is checked as an essential condition for kinematic stability. This saves the computational cost of calculating stiffness matrix for many constraint violating strings.

2.2.1 Sizing Optimal Design

For size optimization, that is, optimization of cross-sectional properties of member, continuous or discrete variables are used corresponding to area of each member in the ground structure.

2.2.2 Topology Optimal Design

Topology optimization is the problem of finding optimal subset of nodes and connectivities between them from the ground structure. No separate variables are used to define the topology of the optimal structures. To denote the presence or absence of a member a cutoff limit is imposed on area of member, and lower bound of those variables are kept less than that cutoff limit as discussed in next chapter.

2.2.3 Configuration Optimal Design

Configuration optimization is the problem of finding optimal nodal location in the space. To achieve configuration optimal design, locations of non-basic nodes are allowed to vary within certain lower and upper limits. So separate continuous variables are used to denote the change in nodal position of each node in x and y directions.

Chapter 3

Proposed Genetic Algorithm

Genetic algorithm is a potential search and optimization procedure which utilizes a blending of principles of natural genetics and natural selection. Genetic algorithm is a population based technique in which population of set of variables are modified in each iteration using some GA operators.

Even though most of GA simulations are performed by binary coding of problem parameters into GA variables, there exists a Real Coded GA where problem parameters are directly used (Deb and Agrawal, 1995). In a real coded GA, a string comprises a set of real values for design variables and population itself consists many such strings. In the present work the cross-sectional areas of member (in most of the problems), and nodal locations are treated as continuous design variables. Since real GA uses continuous problem parameters directly, so real coded GA is used in the present work.

GA differs from traditional optimization in many ways which give GA its relative merit. A few are listed here :

- GA does not require problem specific knowledge to carry out search. GA processes only function value at a given point, not the derivative information, as used in traditional calculus based method.
- GA processes population of points at a time, that is, GA processes number of designs. This peculiarity of GA makes it less susceptible to be

caught in local minima.

- GA uses randomized operators in place of usual deterministic ones. The random operator improves search power in adaptive manner.

3.1 GA Operators

Reproduction crossover and mutation are three main GA operators to simulate biological evolution process. Following is the brief discussion of GA operators used in the present work:

3.1.1 Reproduction

Reproduction constitutes the first step in GA. This operator selects the good string from population and forms a mating pool. *Tournament selection* reproduction operator which is used here, is one of the reproduction operators available in literature.

Tournament selection first picks up s (tournament size) individual strings and puts the best of those in mating pool. Then it selects next s individuals and perform same operation until all individuals in the population are exhausted. The whole population is rearranged randomly and same procedure is repeated until formation of complete mating pool. Thus reproduction operator assigns exactly s copies of best string to mating pool at every generation.

3.1.2 Crossover

Crossover operator proceeds after reproduction in two steps. First two individual strings are selected at random from mating pool and secondly two new strings are created by exchanging information among those strings. Among various crossover operators, simulated binary crossover (SBX), which is used here is found to be particularly useful in problems having multiple optimal solutions with narrow global basin (Deb, and Agrawal, 1994). Number of pairs

of strings actually taking part in the crossover depends upon crossover probability p_c. Since crossover is the main search operator in a GA, p_c is generally kept high.

In SBX two parents p_1 and p_2 are picked up at random from the population and their offsprings c_1 and c_2 are created according to the following equations :

$$c_1 = 0.5(p_1 + p_2) - 0.5\beta(u)|p_2 - p_1|$$
$$c_2 = 0.5(p_1 + p_2) + 0.5\beta(u)|p_2 - p_1|\ .$$

Where $\beta(u)$ is a function of u between 0 to ∞, generated according to predefined probability distribution. This distribution has a property that random numbers near unity are much more probable than far off numbers. To simulate this probability distribution, a random number u between 0 to 1 is created first thereafter $\beta(u)$ is calculated as below (Deb, and Agrawal, 1994) :

$$\beta(u) = \begin{cases} (2u)^{\frac{1}{(n+1)}} & \text{if } u \leq 0.5 \\ \left(\frac{1}{2(1-u)}\right)^{\frac{1}{(n+1)}} & \text{otherwise} \end{cases}$$

The exponent n in the above expression is an important search parameter in SBX. Basically it reflects the propensity of GA to restrict the search to the points that are proximal to initial population. Low values of n corresponds to generating the points far away from parent solutions.

3.1.3 Mutation

Mutation operator takes part in local search around some point. Mutation operator picks up a string from population of strings generated after crossover, and creates a point in vicinity to that. Number of strings actually taking part in mutation depends upon probability of mutation p_m. Since mutation changes the string randomly so p_m generally kept low.

Present mutation operator picks up a string p_1 and at random selects any design variable in that string to change that and create c_1. Design variable x is changed according to following equation :

$$x_{new} = x_{old} + 0.5\beta(u)(x_{max} - x_{min})$$

Where

x_{new} is value of design variable in c_1

x_{old} is value of design variable in p_1

u is a random number according to following (in case of rigid bounds on variables) :

$$\{-1.0 + (1 - \Delta_1)^{(n+1)}\} \leq u \leq \{1.0 - (1.0 - \Delta_2)^{(n+1)}\}$$

Δ_1 and Δ_2 are calculated as follows :

$$\Delta_1 = \frac{2 \times (x - x_{min})}{x_{max} - x_{min}} \qquad \Delta_1 \geq 1.0 \Rightarrow \Delta_1 = 1.0$$

$$\Delta_2 = \frac{2 \times (x_{max} - x)}{x_{max} - x_{min}} \qquad \Delta_2 \geq 1.0 \Rightarrow \Delta_2 = 1.0$$

x_{min} and x_{max} are the lower and upper bounds of particular variable.

$\beta(u)$ is following function of u :

$$\beta(u) = \begin{cases} 1.0 - 2^{\frac{1}{(n+1)}} & \text{if } u \leq -1.0 \\ -\{1.0 - (1.0 + u)^{\frac{1}{(n+1)}}\} & \text{if } -1.0 \leq u \leq 0.0 \\ \{(1.0 - (1.0 - u)^{\frac{1}{(n+1)}}\} & \text{if } 0.0 \leq u \leq 1.0 \\ 1.0 & \text{if } 1.0 \leq u \end{cases}$$

Again exponent n controls proximity of child point to parent point. For higher value of n child will be closer to parent.

3.2 GA Based Problem Formulation

In the context of genetic algorithm, the truss structure optimization problem is posed as follows :

Find $(x_1, x_2 \ldots x_m)$ **to minimize** $F(x)$

subject to $g_j(x) \geq 0$ for $j = 1, 2, \ldots, J$

where $x_1, x_2 \ldots x_m$ represents m design variables corresponding to areas of members in ground structure.
The terms $g_j(x)$ are the set of J constraints. $F(x)$ denotes either to penalty term alone or sum of objective function and penalty term as discussed later.

The objective function is taken as weight of the truss as :

$$f(x) = \sum_{i=1}^{m} x_i l_i \rho_i \qquad \text{where } i \in I$$

$$I = \{i : x_i \geq \epsilon \text{ for } i = 1, 2 \ldots m \}$$

where l_i is the length of i^{th} member in the structure,
ρ_i is the density of material used in i^{th} member,
the ϵ is the cutoff limit on area for deleting members from the ground structure, that is, if $x_i < \epsilon$ then i^{th} member in the ground structure is assumed to be absent.

3.2.1 Fitness Function

Fitness of any string is value returned to GA, based on which GA operators modify the population. In present problem $F(x)$ is used as fitness function which denotes the weight of truss along with penalties. GA operators tend to minimize $F(x)$ which in turn reduces the penalties and the weight.

3.2.2 Variable Bounds

GA generates the variables corresponding to areas of each member in ground structure, within certain lower and upper bounds . These lower and upper bounds along with ϵ are adjusted in such a manner that there are sufficient chances of deletion of member while keeping total range of variable as small as possible. In this way a variable generated by GA corresponding to a particular member in ground structure takes care of both presence or absence of member and area of that member if present. The cutoff limit denoted by ϵ is kept slightly positive below which area of member becomes practically unacceptable.

So lower bound (x_{min}) and upper bound (x_{max}) together with ϵ provides control over topology optimization along-with size optimization. Range (x_{min}, ϵ) in which a member gets deleted is kept 30% to 60% of total range (x_{min}, x_{max}) to obtain desired topology optimization in a few generations.

3.2.3 Constraint Handling

Constraints are handled by incorporating penalty term in function $F(x)$ returned to GA, in the following way :

1. **Kinematic constraints** : Constraints of kinematic stability is imposed in the following steps :

   - Checking whether all basic nodes (nodes with either load or support) are included or not, and assigning high penalty to $F(x)$ in case any basic node is not included. Due to this constraint algorithm keeps all the basic nodes in optimized structure.
   - Checking degree-of-freedom of truss and assigning a high penalty value to $F(x)$ if degree of freedom is greater than zero. This checks the system from becoming a mechanism.
   - Checking determinant of the stiffness matrix and assigning high penalty if found non positive definite. This constraints ensures kinematically stable structure.

   All above kinematic constraints are checked in the above order. Any further calculations are avoided in case of any constraint violation is found at any step. Although checking the positive definiteness of stiffness matrix is sufficient condition, but other two conditions are evaluated prior to that to save computational cost in case of violation.

2. **Displacement constraint** : Displacement constraints keep the displacement of any node of truss, under given loading, within desired limits. Displacement constraint is incorporated by adding ϕ_1 and ϕ_2 terms, proportional to violation of constraint, to the weight, as discussed in the

following :

$$\phi_1 = K \sum_{i=1}^{n} \left| \left\langle \frac{\delta_{max}}{\delta_i^x} - 1 \right\rangle \right|,$$

$$\phi_2 = K \sum_{i=1}^{n} \left| \left\langle \frac{\delta_{max}}{\delta_i^y} - 1 \right\rangle \right|.$$

where

$\langle\ \rangle$ denotes the bracket operator such that $\langle a \rangle$ is equal to a if a is negative, otherwise zero.

δ_i^x and δ_i^y are displacements of i^{th} node in x and y direction respectively.

δ_{max} is the maximum permissible displacement in either x or y direction.

3. **Stress constraints** : This constraint keeps the stresses in each member with in desired limits. Similar to displacement constraint, penalty term ϕ_3 is added to the objective function, in case of violation.

$$\phi_3 = K \sum_{i=1}^{m} \left| \left\langle \frac{\sigma_{max}}{|\sigma_i|} - 1 \right\rangle \right|$$

where:

σ_i is the stress in the i^{th} member.

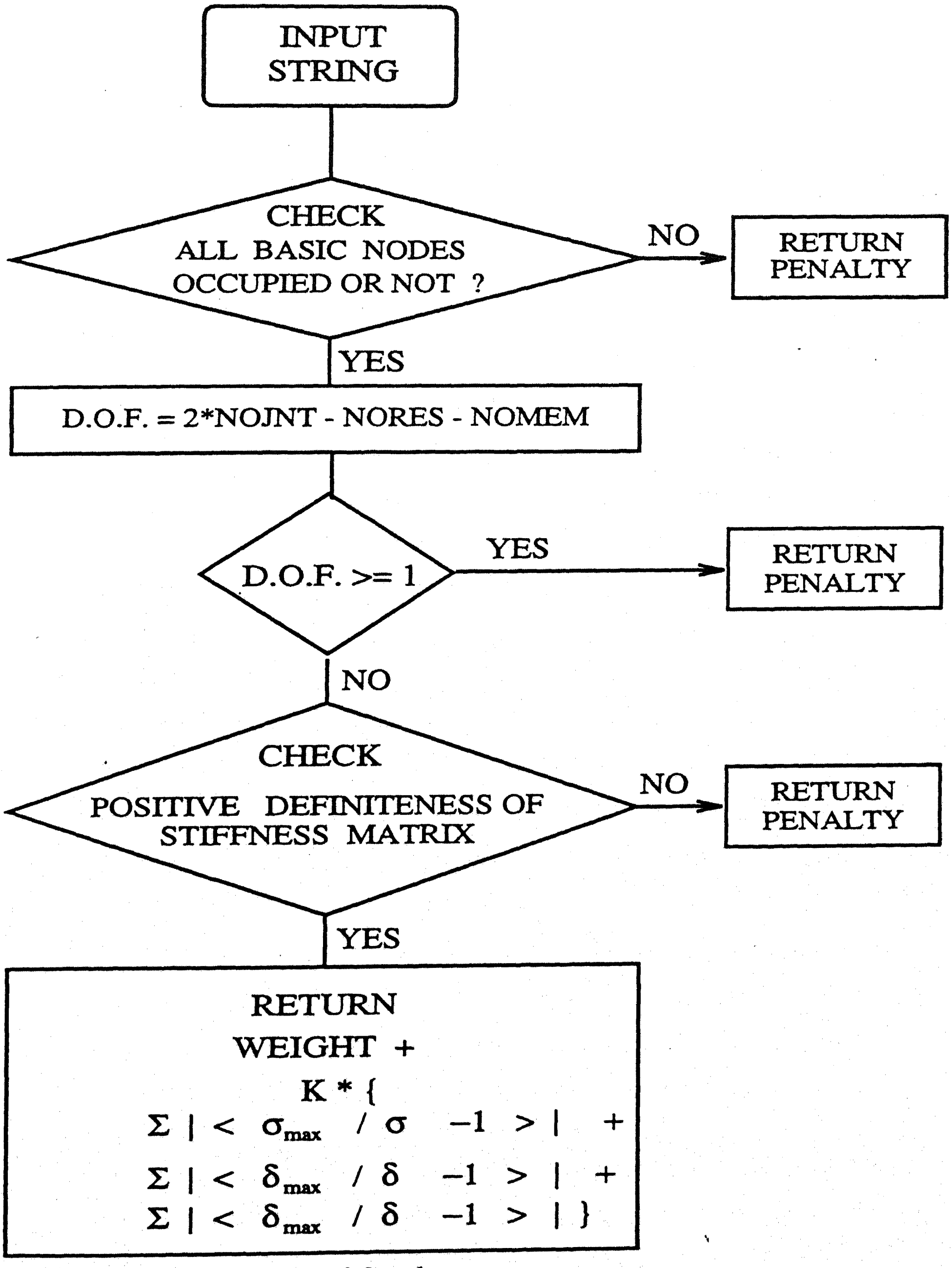
σ_{max} is the maximum permissible stress.

So in case of stress and displacement constraints, $F(x)$ denotes the following :

$$F(x) = f(x) + \phi_1 + \phi_2 + \phi_3$$

Finite element analysis is used to calculate the nodal deflections and member stresses, hence checking the constraint violation.

In the multi-loading case, $F(x)$ are calculated for each loading condition and the highest of the values is returned to GA as the fitness of solution. Figure 3.1 shows the flow chart for calculating fitness of a string.



D.O.F. denotes degree of freedom,
NOJNT is number of joints or nodes in the truss,
NORES is the number of kinematic restraints,
NOMEM is the number of member in the truss.

Figure 3.1: Processing of string in objective function.

3.2.4 Selection of GA Parameters

Selection of GA parameters play important role in getting best results with minimum computational time. These operators are mainly crossover probability (p_c), mutation probability (p_m), population size (N) and exponent n in the SBX operator. Crossover probability controls the number of crossover operations. Since crossover is considered to be the main search operator in a GA (Deb, and Agrawal, 1995), probability of crossover is kept to 0.9. Lower crossover probability does not allow effective search of the whole search space. Second parameter to be fixed is the probability of mutation, which randomly changes the characteristics of any string. Since mutation operator is introduced only to keep diversity in GA population, probability of mutation p_m is kept to be 0.1. These probablities are found to be best by experience and are also similar to those found in earlier works.

Population size in GA controls the number of design to be explored in each generation.Ideal population size is dependent on the problem complexity. In the present problem of minimization of truss weights, the complexity of problem is mainly affected by the number of variables (depends upon the members in the ground structure) and number of basic nodes (depends upon the loading condition). Number of variables directly affects complexity of the search space, so larger number of variables requires larger population size. In problems, presented here, population size is fixed to 10 to 60 times the number of variables. In the case of topology optimization, number of basic nodes also influences the complexity of the optimization problem.

Another important GA parameter is the exponent n which influences the probability distribution in creating children points by exchanging information among parent points in SBX. Larger value of n tends to create the child point closer to the parent. In most problems n is kept fixed to 2. In the problems with discrete probability distribution n is kept varying from 1 to 2.5, which is found to better. This variation of n allows extensive search in initial generations and permits a narrow search in later generations.

Chapter 4

Results

In this chapter, a number of truss structure design problem are borrowed from the literature and the efficiency of proposed algorithm is shown by considering these problems. Following design parameter are kept same to all cases :

Probability of crossover (p_c)	:	0.9
Probability of mutation (p_m)	:	0.1
Tournament size for selection (t_s)	:	2

Population size is varied in problem to problem. However this is kept as $N = cn$, where n is the number of design variables and c is an integer number. Value of c is varied according to complexity of problem.

4.1 Six-Node 2D Truss

First, a six-node 2D truss, commonly used in literature (Hajela, Lee, and Lin, 1993, Chaturvedi, 1995) is used. The ground structure is shown in Figure 4.1. For clarity, overlapping members are shown with a gap in the figure. All 15 member cross-sectional areas are considered as variables. The following design parameters and constraints are used to solve the problem :

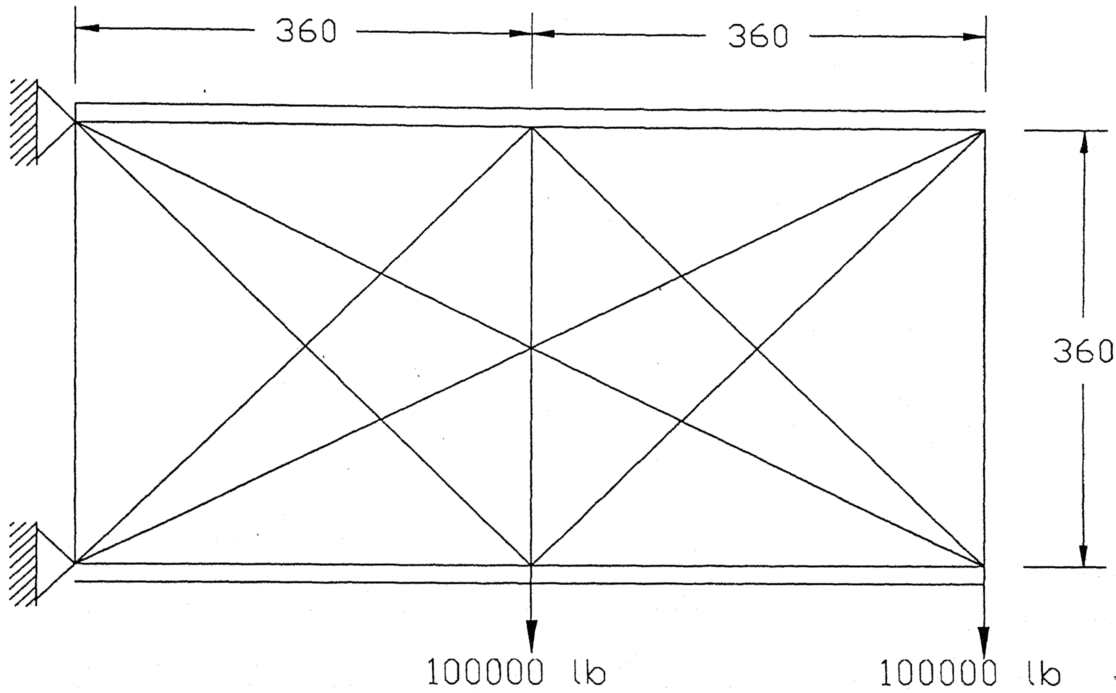


Figure 4.1: Six-node 15-member ground structure (dimensions are in inches).

Youngś modulus of material of members (E)	=	1×10^4 ksi
Maximum permissible compressive stress (σ^c_{max})	=	25 ksi
Maximum permissible tensile stress (σ^t_{max})	=	25 ksi
Maximum permissible displacement		
in either x or y direction	=	2.0 in.
Lower and Upper bounds of areas (x_{min}, x_{max})	=	-35.0 in^2, 35.0 in^2
Density of material (ρ)	=	0.1 lb/in^3
Cutoff limit for area(ϵ)	=	0.09 in^2.

Minimum weight obtained for truss is 4733.44 lb. Topology for the minimum weight truss is shown in Figure 4.2, and member areas for population size 300 and 450, for a run of 225 generations, are shown in Table 4.1.

Six node 2D truss, with only 11 members in the ground structure (shown in Figure 4.3), is again considered keeping all design parameters and loading conditions same as before. Optimal weight in this case (4899.15 lb) is more than that obtained with 15 members in the ground structure (Figure 4.4). This is because longer members are not allowed in this case, one of which is present in the optimum topology in the 15-member case. Optimized truss obtained in

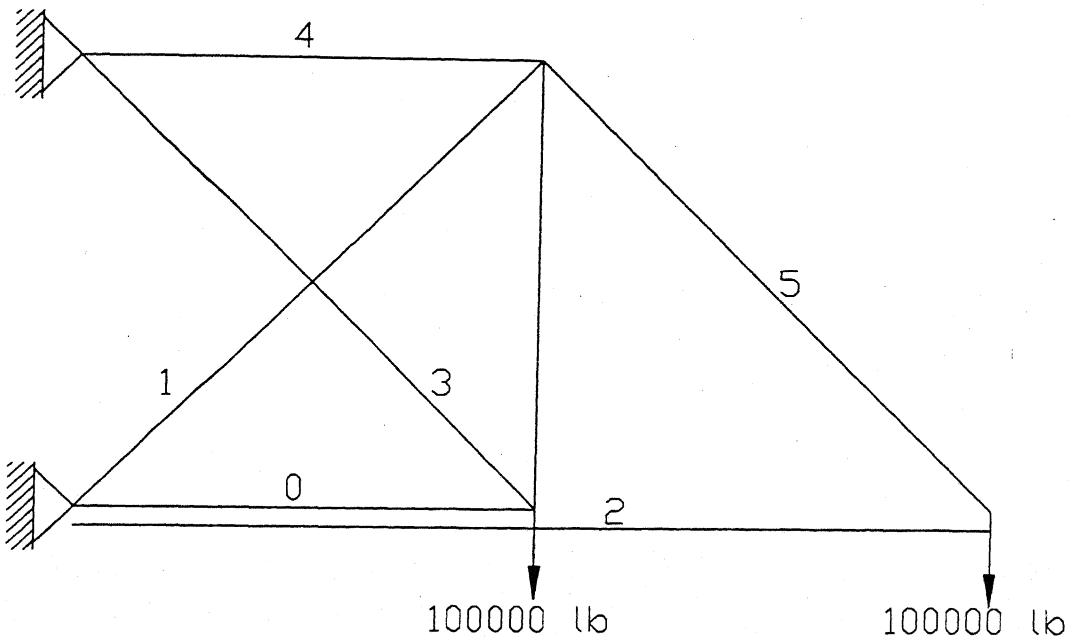


Figure 4.2: Optimized topology for six-node 15-member ground structure.

Member number (refer Figure 4.2)	Area of members (in^2)	
	Population Size 300	Population Size 450
0	05.17	05.22
1	20.05	20.31
2	14.85	14.59
3	07.82	07.77
4	28.29	28.19
5	20.45	20.60
Weight of Truss (lb)	4733.44	4731.65

Table 4.1: Results of six-node 2D truss with 15-members ground structure.

this case is shown in Figure 4.4, and the optimum member areas for population 110 and 220 (225 generations) are listed in Table 4.2.

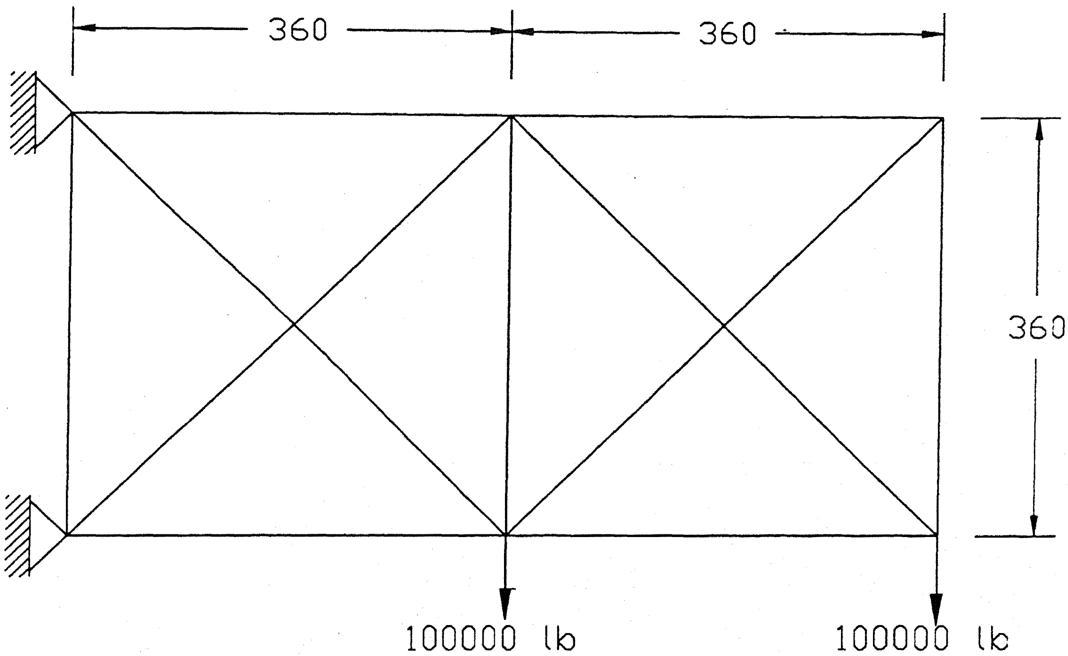


Figure 4.3: Six-node 11-member ground structure (dimensions are in inches).

Above six-node problem is further considered with both 15-member and 11-member ground structures but only discrete member areas are allowed. Two cases of discrete are considered, area having an increment of 1.0 in^2 and area having an increment of 0.1 in^2. Discrete probability distribution is used to produce discrete areas of members, in SBX. Optimum truss weights for these cases are listed in Table 4.3. This discrete area problem is also considered with only stress constraints both with 15-member and 11-member ground structure. Comparison of weight obtained in different cases of present study, along-with that obtained in the previous studies is shown in Table 4.3. In case of both stress and displacement constraints weight of the optimized truss with 15-member ground structure having continuous areas is smaller. With the discrete areas, optimized weight of the truss is more and it is higher for the cases with greater step size. Weight of truss with 11-member ground structure, in all cases, is more than that with the corresponding cases of the 15-member ground

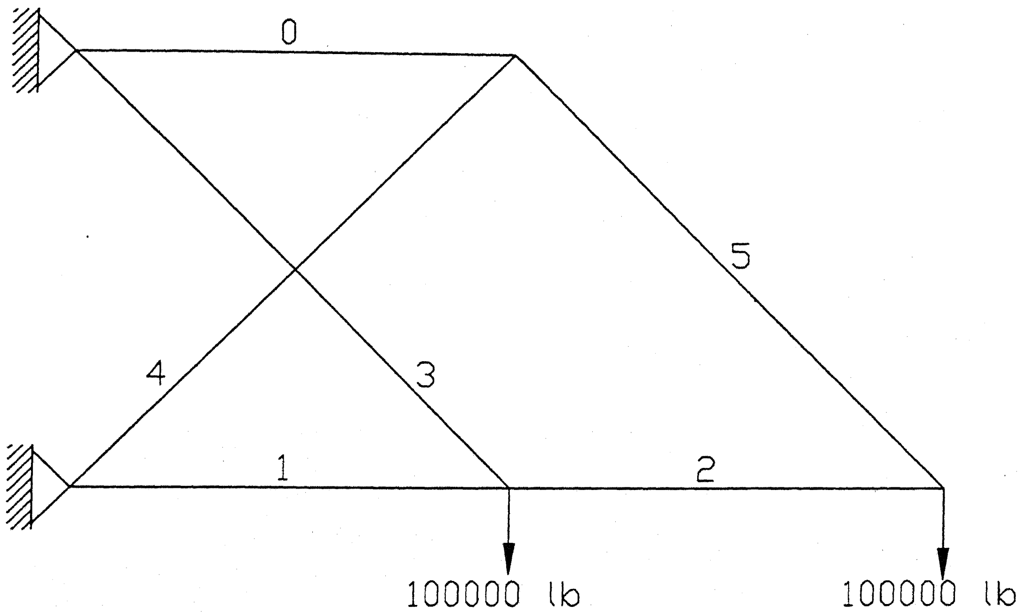


Figure 4.4: Optimized topology for six-node 11-member ground structure.

Member number (refer Figure 4.4)	Area of members (in^2)		
	Proposed		Ringertz (1985)
	Population Size 110	Population Size 220	
0	33.66	29.68	30.10
1	23.12	22.07	22.00
2	16.74	15.30	15.00
3	06.09	06.09	06.08
4	18.55	21.44	21.30
5	20.61	21.29	21.30
Weight of Truss (lb)	4950.75	4899.15	4900.00

Table 4.2: Results of six-node 2D truss with 11-members ground structure.

structure. Weight obtained in the present study compares favorably with those reported in the literature. Comparison of areas of members, obtained in the 15-member ground structure case, with those obtained in previous studies are shown in Table 4.2 and Table 4.4 for continuous areas and discrete areas respectively. These tables show that areas obtained in present study are better than those reported in the literature. So far best known solution in the literature are of Ringertz (1985) in continuous area case, and of Hajela, Lee, and Lin (1993) for discrete area case.

	Weight of Truss (lb)	
	Proposed	Hajela, Lee and Lin (1993)
Continuous areas		
15-member ground structure	4731.7	-
11-member ground structure	4899.2	-
Discrete areas ($\Delta = 1.0$ in^2)		
stress and displacement constraints		
15-member ground structure	4819.8	-
11-member ground structure	4912.9	4942.7
only stress constraints		
15-member ground structure	1636.4	-
11-member ground structure	1636.4	1636.4
Discrete areas ($\Delta = 0.1$in^2)		
stress and displacement constraints		
15-member ground structure	4733.4	-
11-member ground structure	4901.0	-

Table 4.3: Comparision of results of six-node truss

Figure 4.5 shows the best weight of truss with the progress of generations for 15-member ground structure, for 3 different initial populations. In all these cases, initially the weight of truss is different but converges to the same solution. Figure 4.6 and Figure 4.7 shows optimization trend for different population sizes, for 15 member and 11 member ground structures respectively. These figures show that with higher population size convergence is faster than with lower population size.

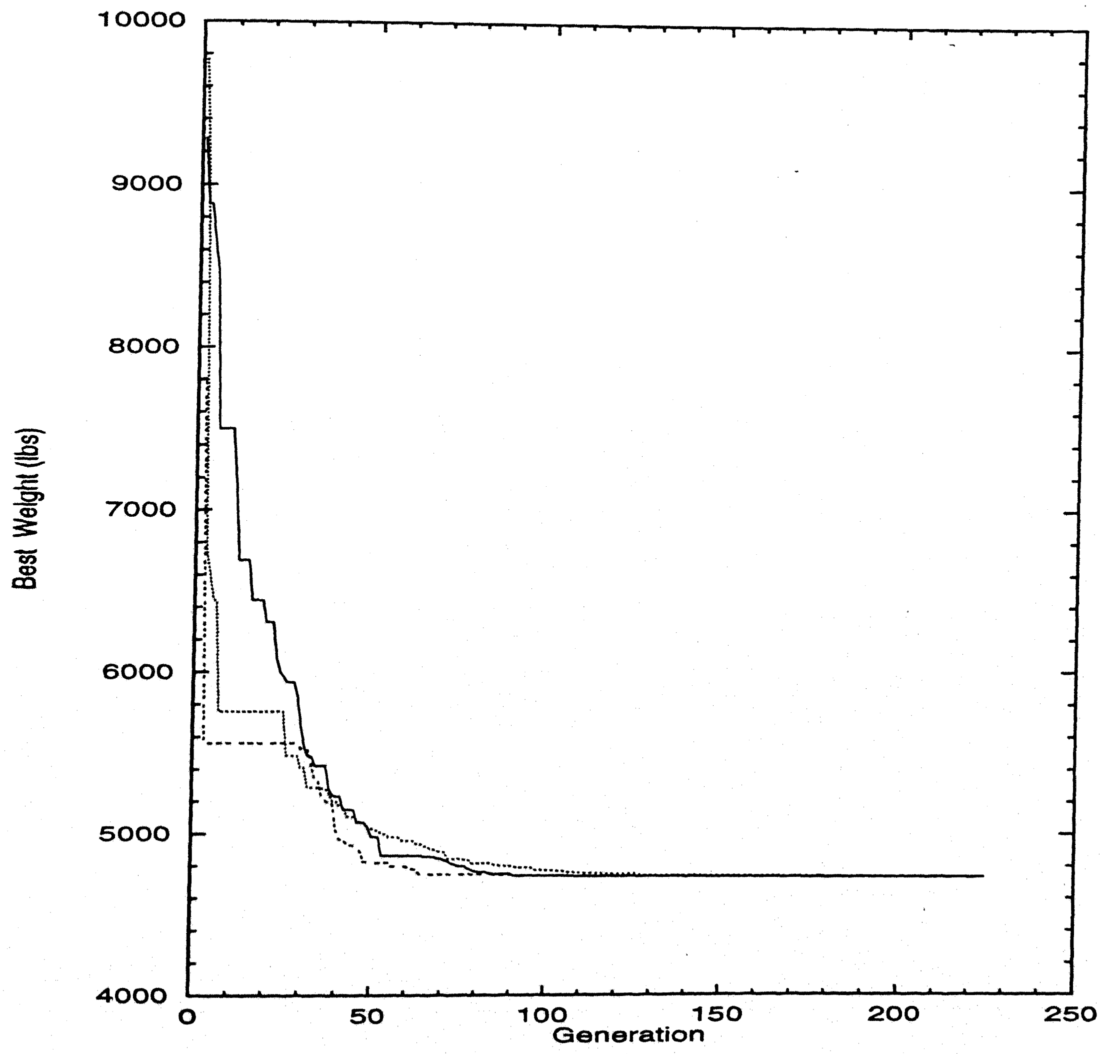


Figure 4.5: Optimization of six-node 15-member ground structure using 3 different initial populations.

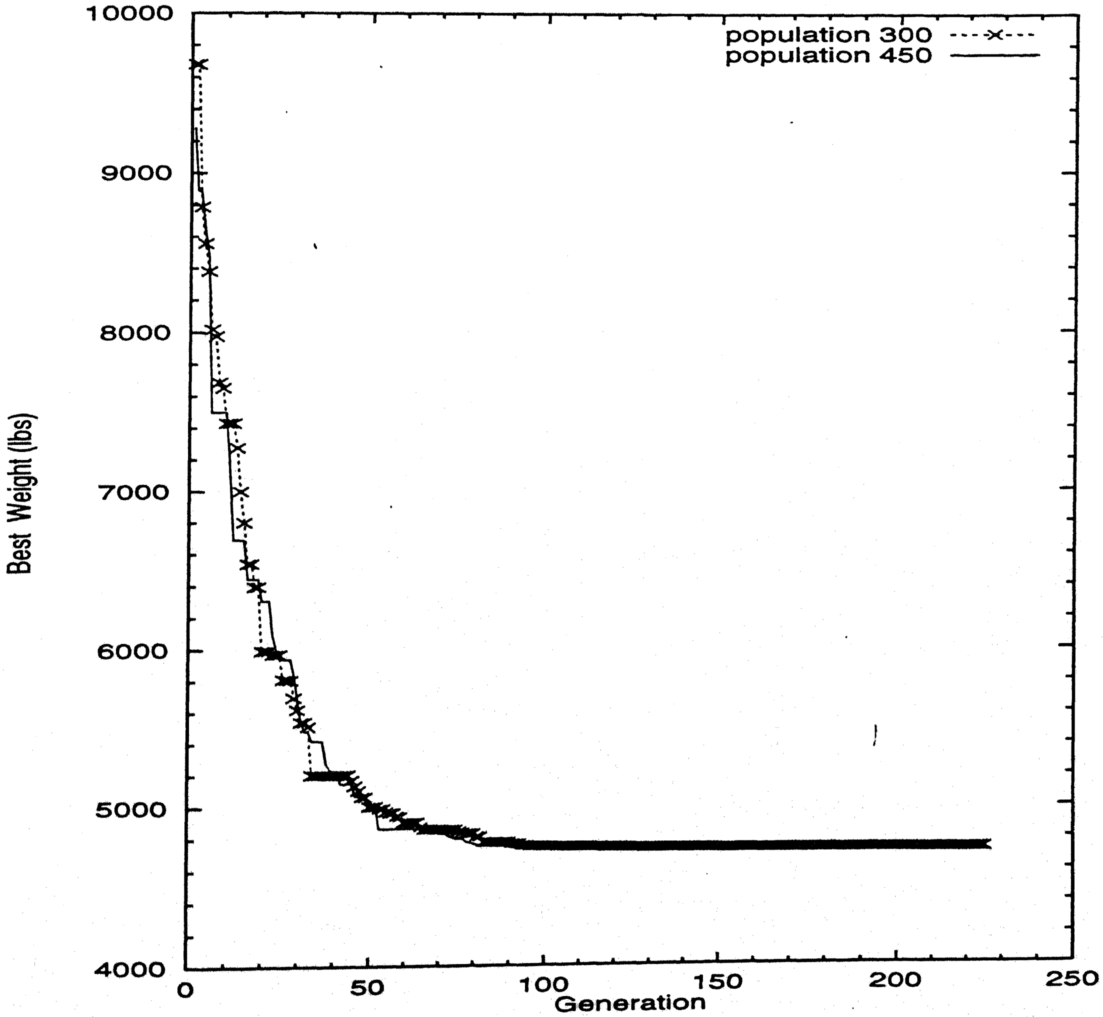


Figure 4.6: Optimization of the six-node 15-member ground structure.

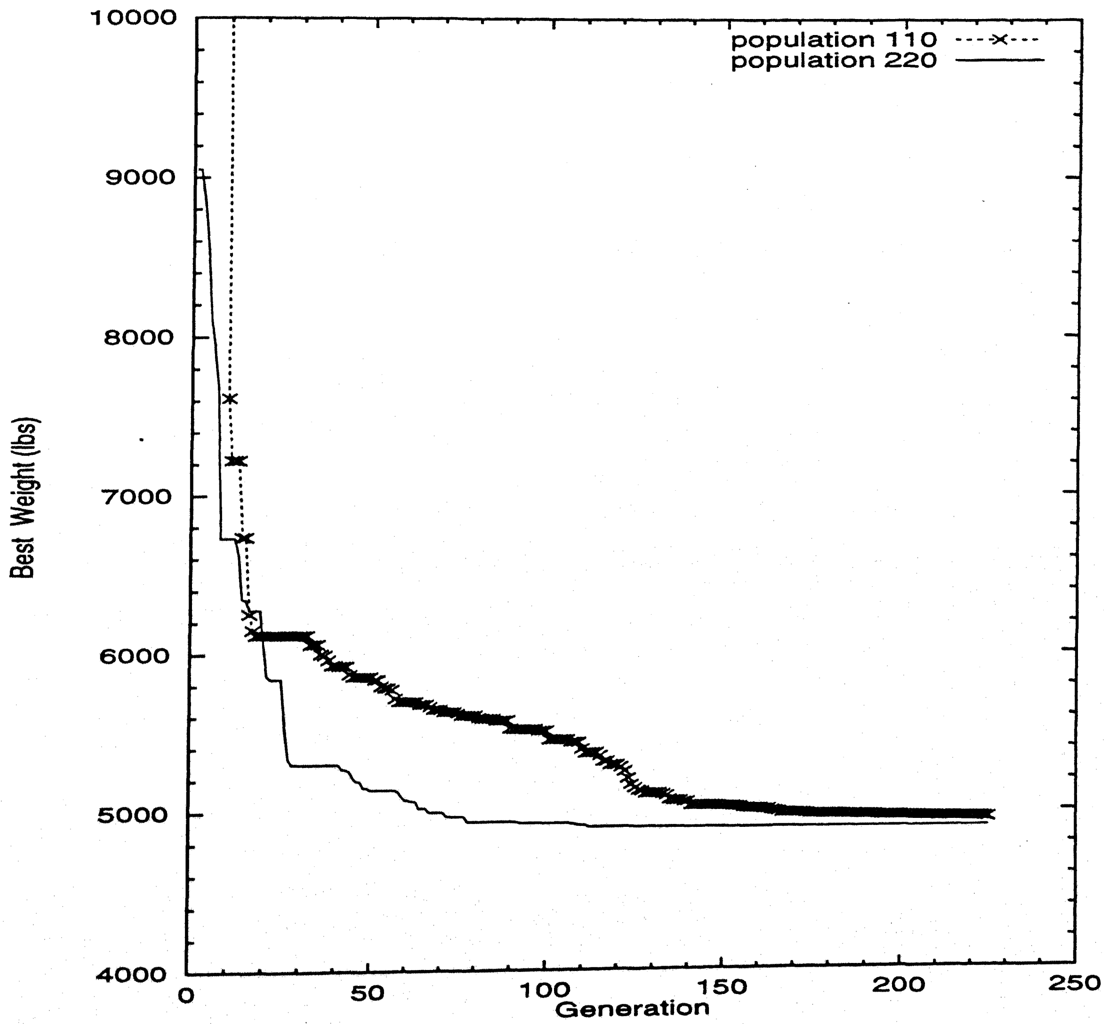


Figure 4.7: Optimization of the six-node 11-member ground structure.

Member number (refer Figure 4.4)	Areas of member (step size = 1.0 in^2)	
	Proposed	Hajela, Lee, and Lin (1993)
0	30	28
1	24	24
2	16	16
3	6	6
4	20	21
5	21	22
Weight of truss (in lb)	4912.85	4942.7

Table 4.4: Comparision of areas of six-node truss with 11-member ground structure.

4.2 Ten-Node 2D Truss

Ten-node 2D simply supported truss with all 45 members in the ground structure (node positions shown in Figure 4.8) is considered with the following design parameters:

Young's modulus of material of members (E)	=	1×10^4 ksi
Maximum permissible compressive stress (σ^c_{max})	=	25 ksi
Maximum permissible tensile stress (σ^t_{max})	=	25 ksi
Maximum permissible displacement in either x or y direction	=	2.0 in.
Lower and Upper bounds of areas (x_{min}, x_{max})	=	-1.0 in^2, 1.0 in^2
Density of material (ρ)	=	0.1 lb/in^3
Cutoff limit for area (ϵ)	=	0.02 in^2.

This 45 variable truss problem is solved with population size of 1800. Optimal topology of truss (weight 44.03 lb) is shown in Figure 4.8, and member areas are listed in Table 4.5. Here although variables are not grouped to impose symmetry, still optimized topology is symmetric due to symmetric loading

conditions. Since this problem is not reported in literature so comparisons are not presented. However, the obtained optimized truss seems to have eliminated 5 of 10 nodes and obtained a intuitively optimal structure.

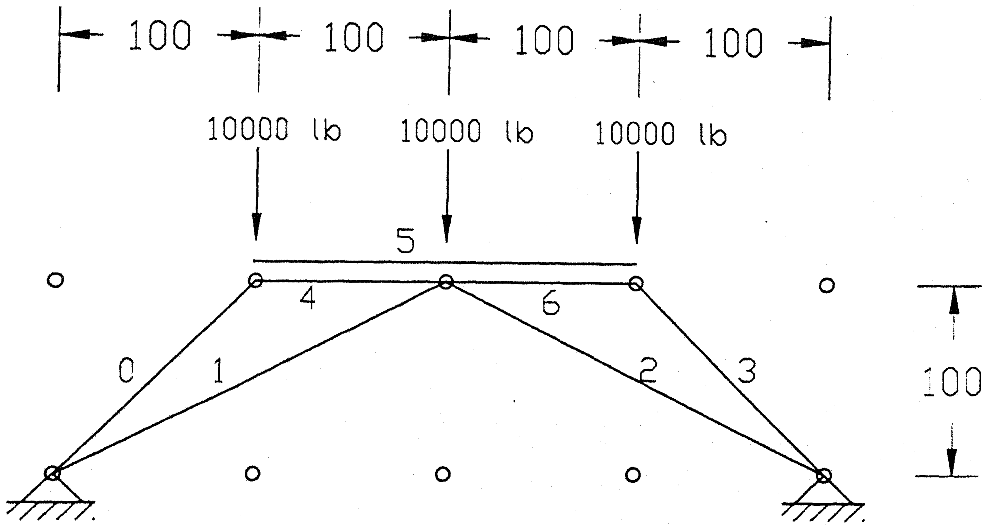


Figure 4.8: Optimized topology and node location for ten-node 2D truss (dimensions are in inches).

Member number (refer Figure 4.8)	Areas of members (in^2)
0	0.566
1	0.477
2	0.477
3	0.566
4	0.082
5	0.321
6	0.080
Weight of truss (lb)	44.033

Table 4.5: Results of ten-node 2D truss

4.3 Two-Tier 2D Truss

Two tier 2D truss with 61-members ground structure (Figure 4.9) is considered for simultaneous topology and sizing optimization. Here also overlapping members are shown with a gap in the figure. Continuous variables with appropriate bounds along-with the cutoff limit are used to obtain both topology and size optimization. Symmetry of truss about middle vertical member is assumed, thus reducing the number of variables to 33. Following parameters are used in the problem :

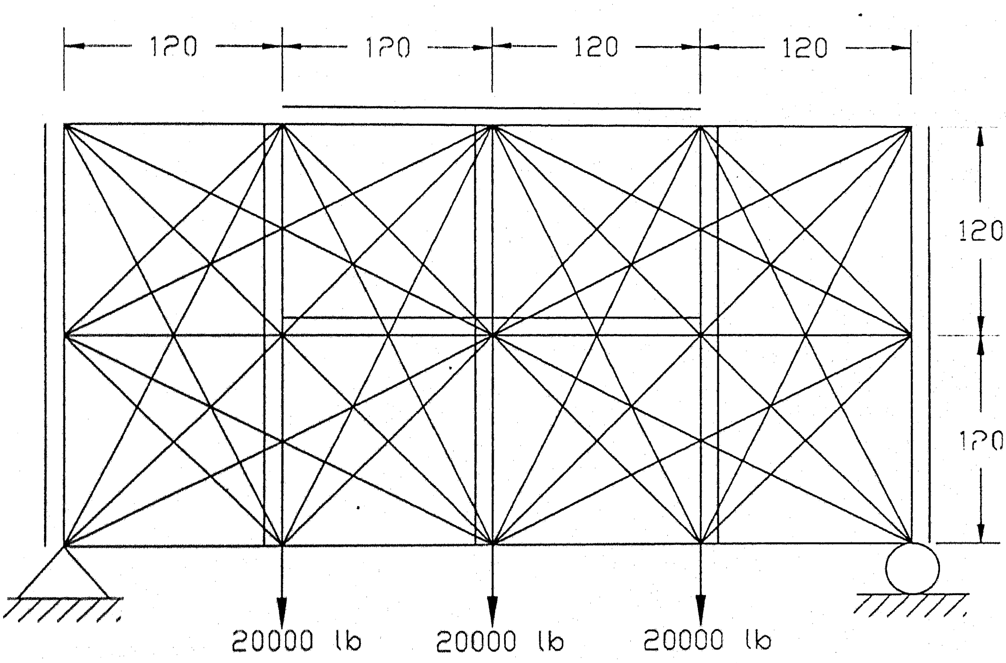


Figure 4.9: Two-tier 61-member ground structure (dimensions are in inches).

Young's modulus of material of members (E)	=	1×10^4 ksi
Maximum permissible compressive stress (σ^c_{max})	=	20 ksi
Maximum permissible tensile stress (σ^t_{max})	=	20 ksi
Maximum permissible displacement in either x or y direction	=	2.0 in.
Lower and Upper bounds of areas (x_{min}, x_{max})	=	-2.25 in^2, 2.25 in^2
Density of material(ρ)	=	0.1 lb/in^3
Cutoff limit for area (ϵ)	=	0.05 in^2

Various runs are taken for this problem and simulations are continued till 250 generations. Two typical topologies are shown in Figure 4.10 (weight 207.869 lb) and Figure 4.11 (weight 198.485 lb) for population sizes 1320 and 1650, respectively. Areas of members for these topologies are shown in Table 4.6 and Table 4.7. In the first case, only 20 members out of 61 are required in the optimal truss and 4 nodes out of 15 are eliminated. In the second case, although the overall weight is less than that in first case, 19 members are required and 3 nodes are eliminated.

Sizing and topology optimization is again carried out for 61-member ground structure with discrete areas. Discrete probability distribution in SBX is used to generate areas with step size of 0.15 in^2. Weight of the truss obtained in this case is 227.58 lb which is less than that obtained by Hajela, Lee and Lin (1993), that is 235.00, with the same step size. Since topologies obtained in the present study and that of literature are different so area-wise comparison are not presented.

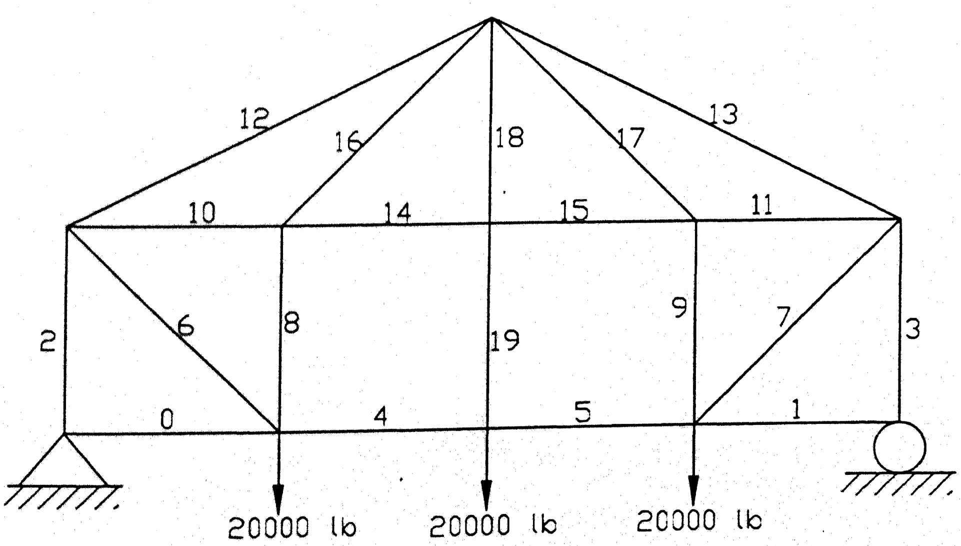


Figure 4.10: Optimized topology for two-tier 61-member ground structure ($N = 1320$).

Member number (refer Figure 4.10)	Areas of members (in^2)
0,1	0.054
2,3	1.511
4,5	1.084
6,7	1.491
8,9	0.081
10,11	0.056
12,13	1.105
14,15	0.136
16,17	0.091
18	1.059
19	1.012
Total Weight of Truss (in lb)	207.869

Table 4.6: Results for two-tier 2D truss with 61-member ground structure ($N = 1320$).

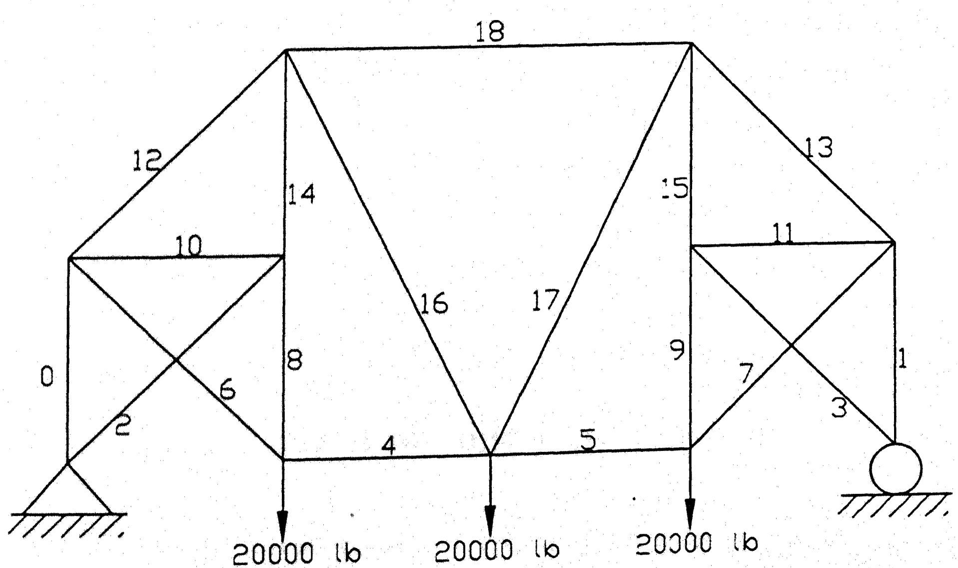


Figure 4.11: Optimized topology for two-tier 61-member ground structure ($N = 1650$).

Member number (refer Figure 4.11)	Areas of members (in^2)
0,1	1.503
2,3	0.064
4,5	0.763
6,7	1.080
8,9	0.251
10,11	0.061
12,13	1.074
14,15	0.256
16,17	0.570
18	1.024
Total Weight of Truss (in lb)	198.485

Table 4.7: Results for two-tier 2D truss with 61-member ground structure ($N = 1650$).

To reduce the complexity of problem, the above ground structure shown in Figure 4.9 is simplified by removing 3 apparently less important nodes(member connected to these nodes are not appeared in optimal topology for 61-member case) and their corresponding members. Ground structure thus reduced to 39 members (Figure 4.12), and number of variables (again considering symmetry) reduced to 21. Two classes of optimization is used, simultaneous sizing and topology optimization and then simultaneous optimization of sizing, topology, and configuration.

4.3.1 Sizing and Topology Optimization

Simultaneous optimization of topology and sizing is carried out taking 21 continuous variables corresponding to 39 members after considering symmetry. Two best topologies for population sizes 630 and 840 (250 generations) are shown in Figure 4.13 and Figure 4.14 respectively. Weight of these topologies are 198.035 lb and 196.546 lb respectively. Member areas for these topologies are listed in Table 4.8 and Table 4.9. In the first case (population size 630) optimized topology has 17 members, and 2 nodes are eliminated from the

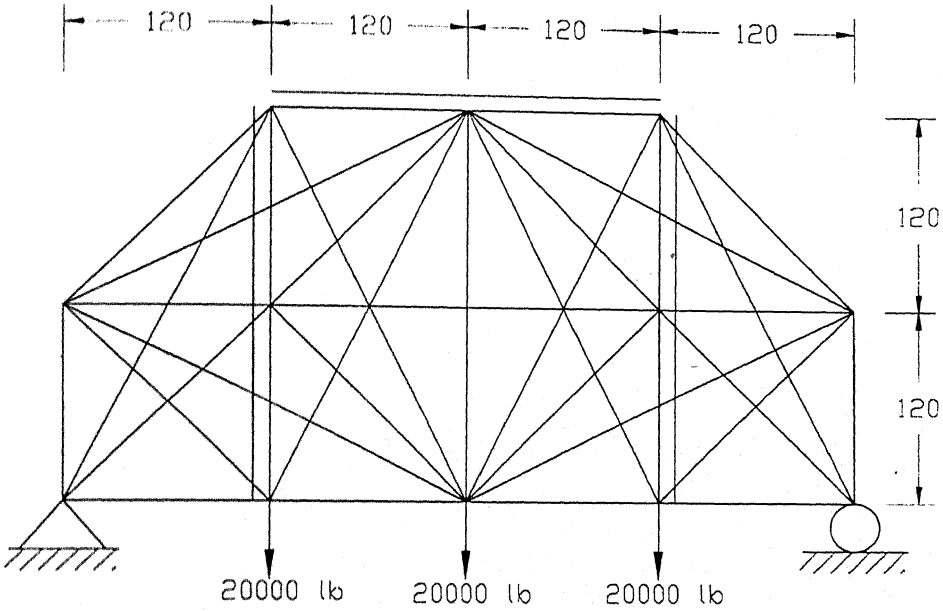


Figure 4.12: Two-tier 39-member ground structure (dimensions are in inches).

ground structure. The optimal topology in second case has 19 members and only 1 node is eliminated from the ground structure.

Minimum weight for 39-member ground structure (196.546 lb) obtained using sizing and topology optimization is even less than that for 61-member ground structure (198.485 lb), with much less population size . Above results show that with change in the population size different truss topologies are obtained having small difference in total weight. This may be due to the large number of solutions that may exist with almost same weight.

Reduction of optimization of weight with generations, for continuous areas, are shown in Figure 4.15 and Figure 4.16 for 61-member and 39-member ground structures respectively. Again with higher population size convergence is faster. Both figures show that convergence in initial few generations is faster and gradually becomes slower.

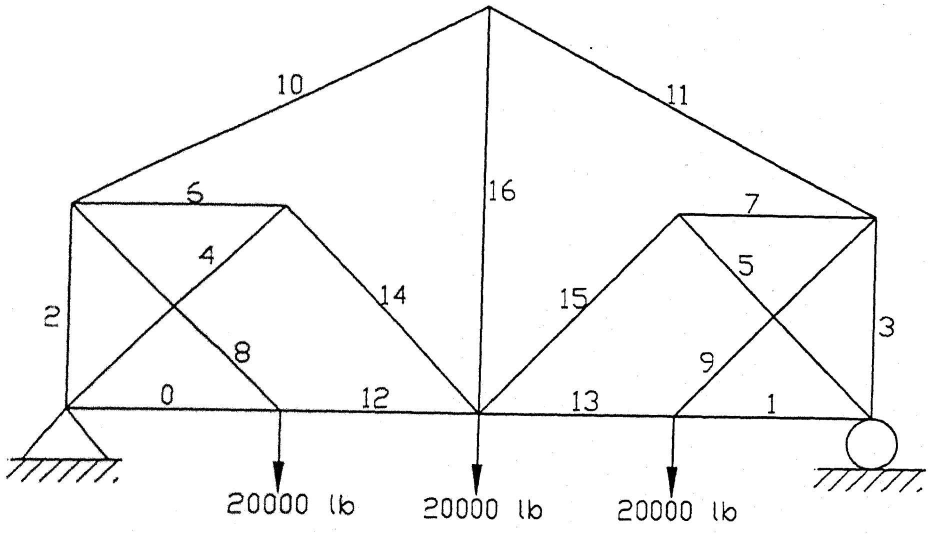


Figure 4.13: Optimized topology for two-tier 39-member ground structure, obtained using sizing and topology optimization ($N = 630$).

Member number (refer Figure 4.13)	Areas of members (in^2)
0,1	0.050
2,3	1.501
4,5	0.052
6,7	0.050
8,9	1.416
10,11	1.118
12,13	1.001
14,15	0.050
16	1.002
Total Weight of Truss (in lb)	198.035

Table 4.8: Results for two-tier 2D truss with 39-member ground structure, obtained using sizing and topology optimization ($N = 630$).

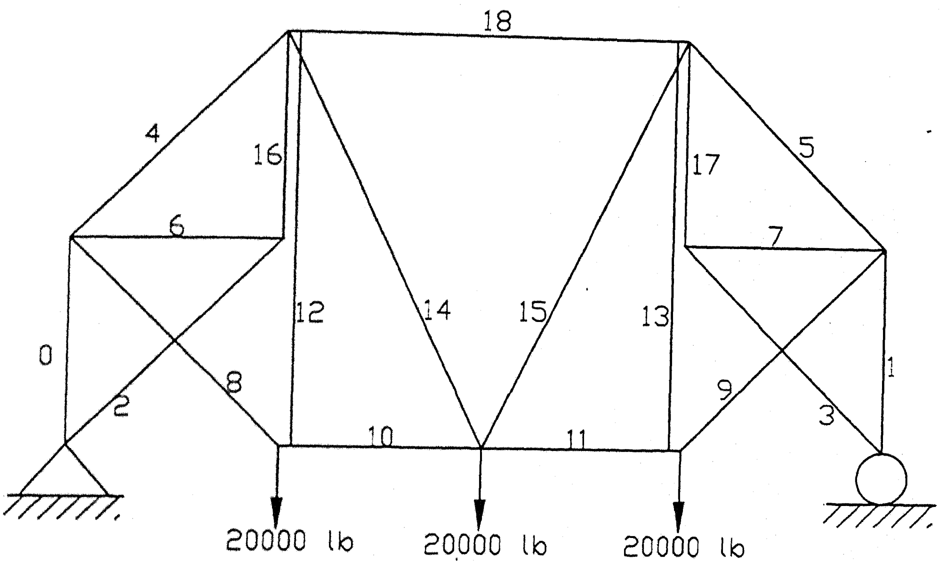


Figure 4.14: Optimized topology for two-tier 39-member ground structure, obtained using sizing and topology optimization ($N = 840$).

Member number (refer Figure 4.14)	Areas of members (in^2)
0,1	1.502
2,3	0.051
4,5	1.063
6,7	0.051
8,9	1.061
10,11	0.751
2,13	0.251
14,15	0.559
16,17	0.052
18	1.005
Total Weight of Truss (in lb)	196.546

Table 4.9: Results for two-tier 2D truss with 39-member ground structure, using sizing and topology optimization ($N = 840$).

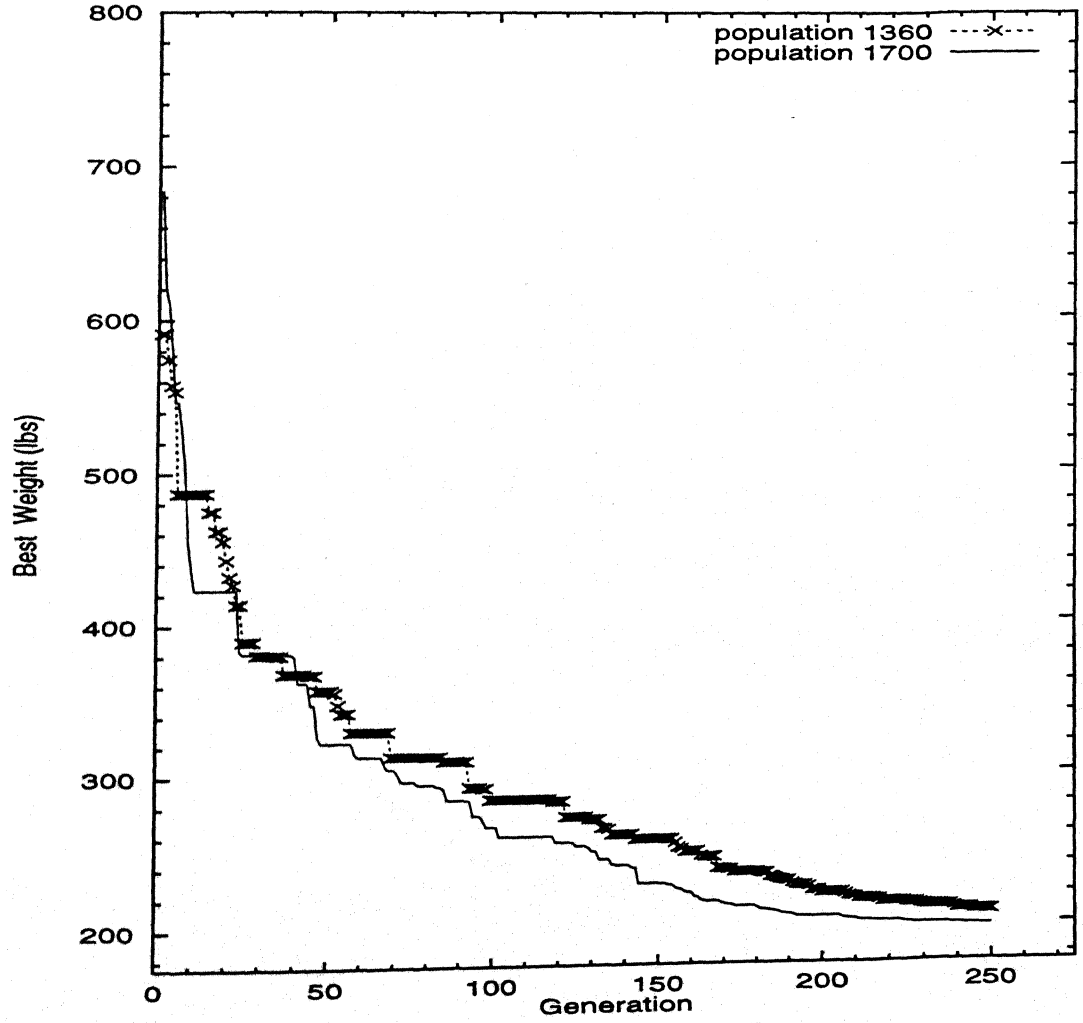


Figure 4.15: Optimization of two-tier 61-member ground structure.

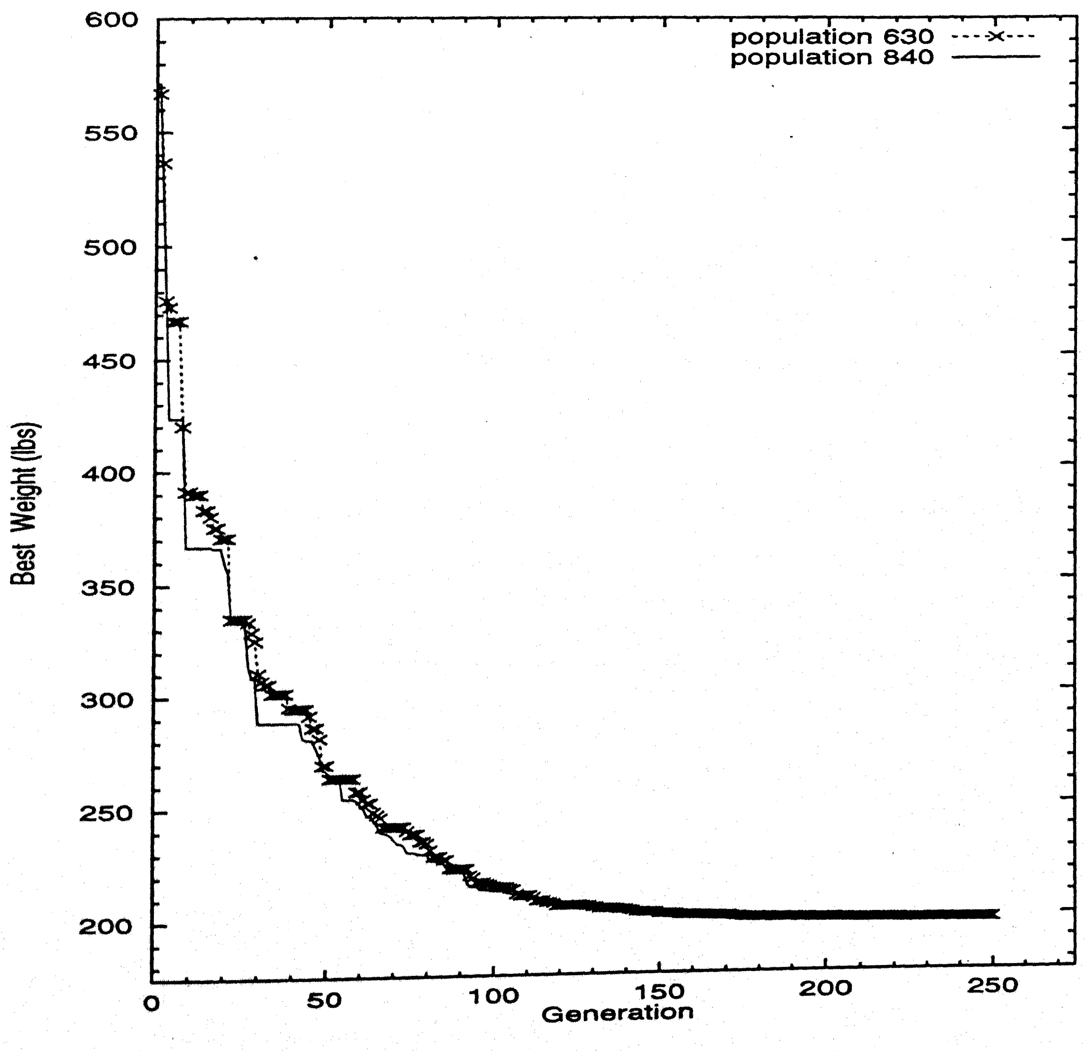


Figure 4.16: Optimization of two-tier 39-member ground structure using sizing and topology optimization.

4.3.2 Sizing, Topology and Shape Optimization

For simultaneous optimization of sizing, topology, and configuration, 7 extra continuous variables in addition to those 21 discussed previously, are considered. Nodal displacement of non-basic nodes in x and y direction, with respect to their original position in ground structure (keeping symmetry) are denoted by these 7 variables. Following extra design parameter is used :

Lower and Upper bounds of nodal displacement in either x or y direction = −120 in, 120 in

Optimized nodal configuration and topology corresponding to a GA run with 1680 population size (300 generation) is shown in Figure 4.17, and optimized member areas are listed in Table 4.10. In this case, number of variables required is more than that required in only topology and sizing optimization for the same ground structure. Thus population size requirement is also high. Optimized topology in this case consists of only 15 members, compared to 19 in the case of sizing and topology optimization.

Member number (refer Figure 4.17)	Areas of members (in^2)
0,1	0.595
2,3	1.615
4,5	1.293
6,7	1.155
8,9	0.051
10,11	1.166
12,13	0.504
14	1.358
Total Weight of Truss (in lb)	192.192

Table 4.10: Results for combined size, shape, and topology optimization for two-tier 2D truss with 39-member ground structure.

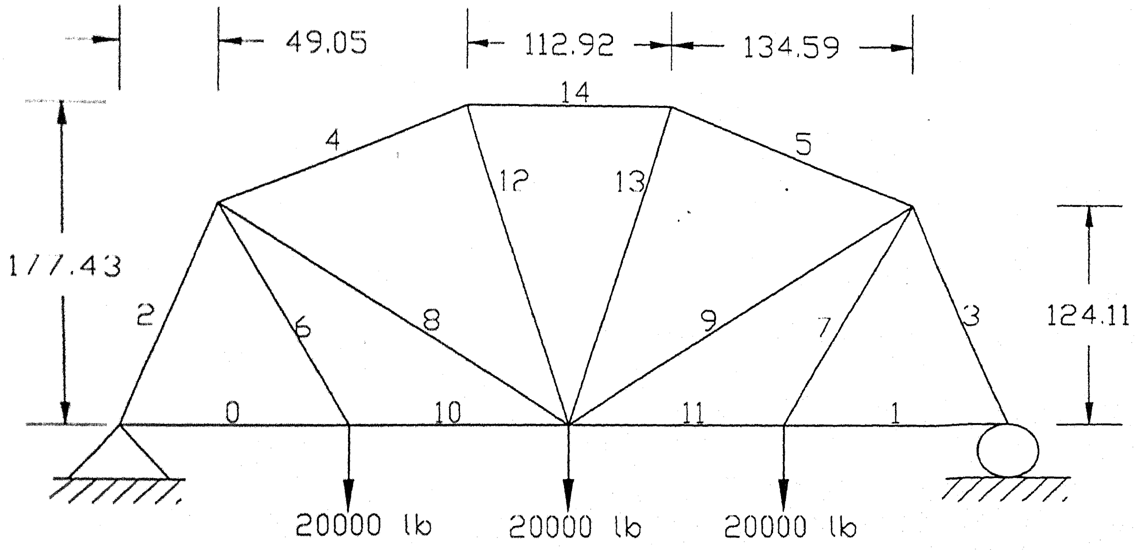


Figure 4.17: Optimized topology and configuration for two-tier 39-member ground structure, obtained using sizing, shape and topology optimization (optimized topology is not a two tier-truss).

4.4 Ten-Node 3D Truss

Ground structure for 10 node 3D truss which consists of 25 members, taken from the literature (Haug and Arora, 1989) is shown in Figure 4.18. In this thesis, this structure is used for (1) Only area optimization and (2) Topology and area optimization. The following parameters are used:

Young's modulus of material of members (E)	=	1×10^4 ksi
Maximum permissible compressive stress (σ^c_{max})	=	40 ksi
Maximum permissible tensile stress (σ^t_{max})	=	40 ksi
Maximum permissible displacement in either x, y or z direction	=	0.35 in.
Density of material (ρ)	=	0.1 lb/in^3

Members are grouped considering the symmetry on opposite sides and cross-members to be symmetric on all the sides, thus reducing the number of variables to 7. This grouping is done in the following way (refer Figure 4.18) :

Group 0: A_0

Group 1: A_1, A_2, A_3, A_4

Group 2: A_5, A_6, A_7, A_8

Group 3: $A_9, A_{10}, A_{11}, A_{12}$

Group 4: $A_{13}, A_{14}, A_{15}, A_{16}$

Group 5: $A_{17}, A_{18}, A_{19}, A_{20}$

Group 6: $A_{21}, A_{22}, A_{23}, A_{24}$

This truss is optimized for two separate loading conditions discussed in Table 4.11, and used in Haug and Arora, (1989).

Node No	F_x	F_y	F_z
	(lb)	(lb)	(lb)
Loading condition 1			
1	0.0	20000.0	-5000.0
2	0.0	-20000.0	-5000.0
3	0.0	0.0	0.0
6	0.0	0.0	0.0
Loading condition 2			
1	1000.0	10000.0	-5000.0
2	0.0	10000.0	-5000.0
3	500.0	0.0	0.0
6	500.0	0.0	0.0

Table 4.11: Loading for 3D 25-member and 39-member ground structure.

4.4.1 Size Optimization

For only size optimization the lower bound of area (x_{min}) is kept larger than the cutoff limit (ϵ) to avoid deletion of members.

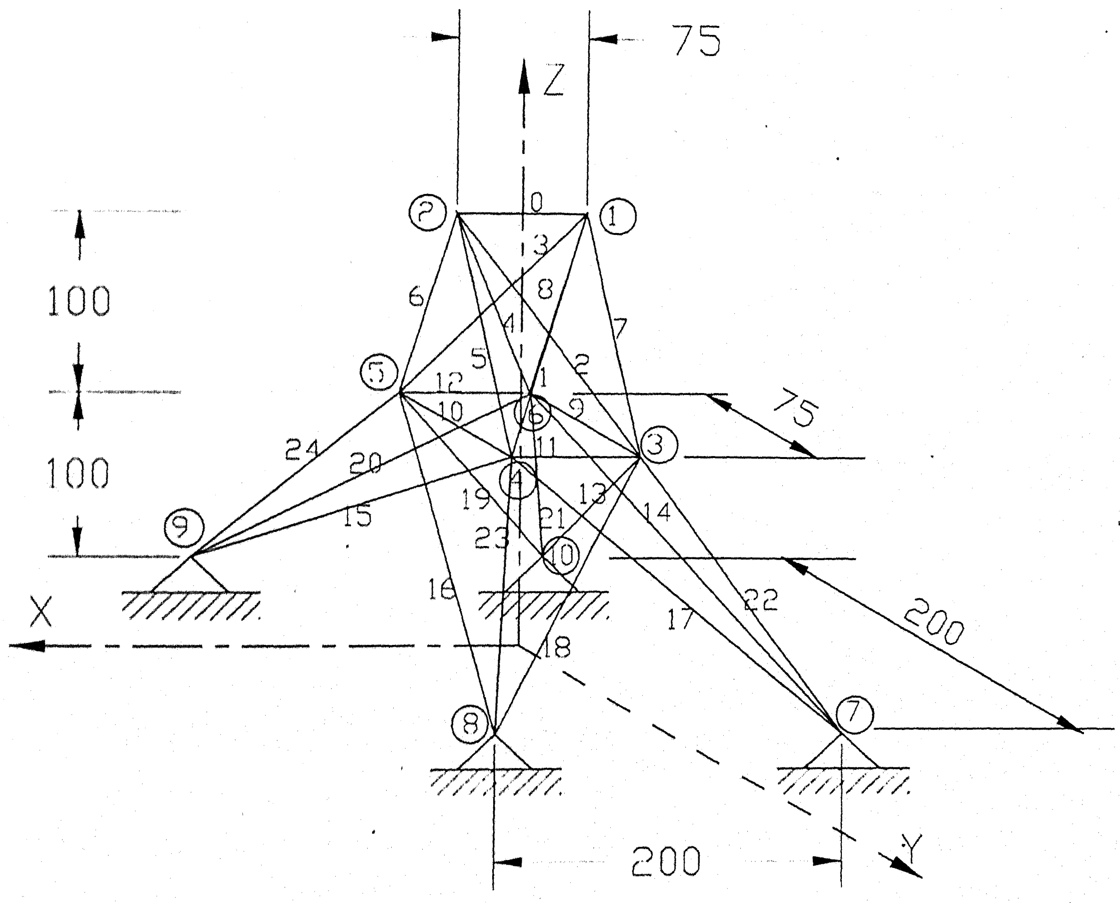


Figure 4.18: 3D 25-member ground structure (dimensions are in inches).

Lower and Upper bounds of areas (x_{min}, x_{max})	=	0.001 in^2, 3.00 in^2
Cutoff limit for area (ϵ)	=	0.00 in^2

Optimized weight obtained for the truss is 544.98 lb. Results of a typical run with a population size of 140 and comparison with those obtained by Haug, and Arora (1989) are shown in Table 4.12. Our results are more or less same as that reported in the literature. The truss satisfies all constraints corresponding to both loading conditions.

Members number (refer Figure 4.18)	Areas of member (in^2)	
	Proposed Population size 210	Haug, and Arora, (1989)
0	0.006	0.010
1,2,3,3	2.092	2.048
5,6,7,8	2.884	2.997
9,10,11,12	0.001	0.010
13,14,15,16	0.690	0.685
17,18,19,20	1.640	1.622
21,22,23,24	2.691	2.671
Weight of Truss(in lb)	544.984	545.050

Table 4.12: Sizing optimization results for 3D 25-member ground structure.

4.4.2 Size and Topology Optimization

For simultaneous size and topology optimization, the variable bounds are adjusted as follows :

$$\begin{aligned}
\text{Lower and Upper bounds of areas } (x_{min}, x_{max}) &= -3.00 \text{ in}^2,\ 3.00 \text{ in}^2 \\
\text{Cutoff limit for area}(\epsilon) &= 0.005 \text{ in}^2
\end{aligned}$$

Minimum weight topology obtained using GA with 20 members is shown in Figure 4.19. Member areas for typical runs with population size of 70 and 140 are listed in Table 4.13. Although the minimum weight of 545.167 lb obtained in this case is not less than that obtained in the previous case (only size optimization), but 5 less members are required in this truss.

Memebr number (refer Figure 4.19)	Areas(in^2)	
	Population size 70	Population size 140
0,1,2,3	2.066	2.161
4,5,6,7	2.809	2.858
8,9,10,11	0.790	0.721
12,13,14,15	1.700	1.596
16,17,18,19	2.594	2.667
Weight of truss (in lb)	546.683	545.167

Table 4.13: Sizing and topology optimization results for 3D 25-member ground structure.

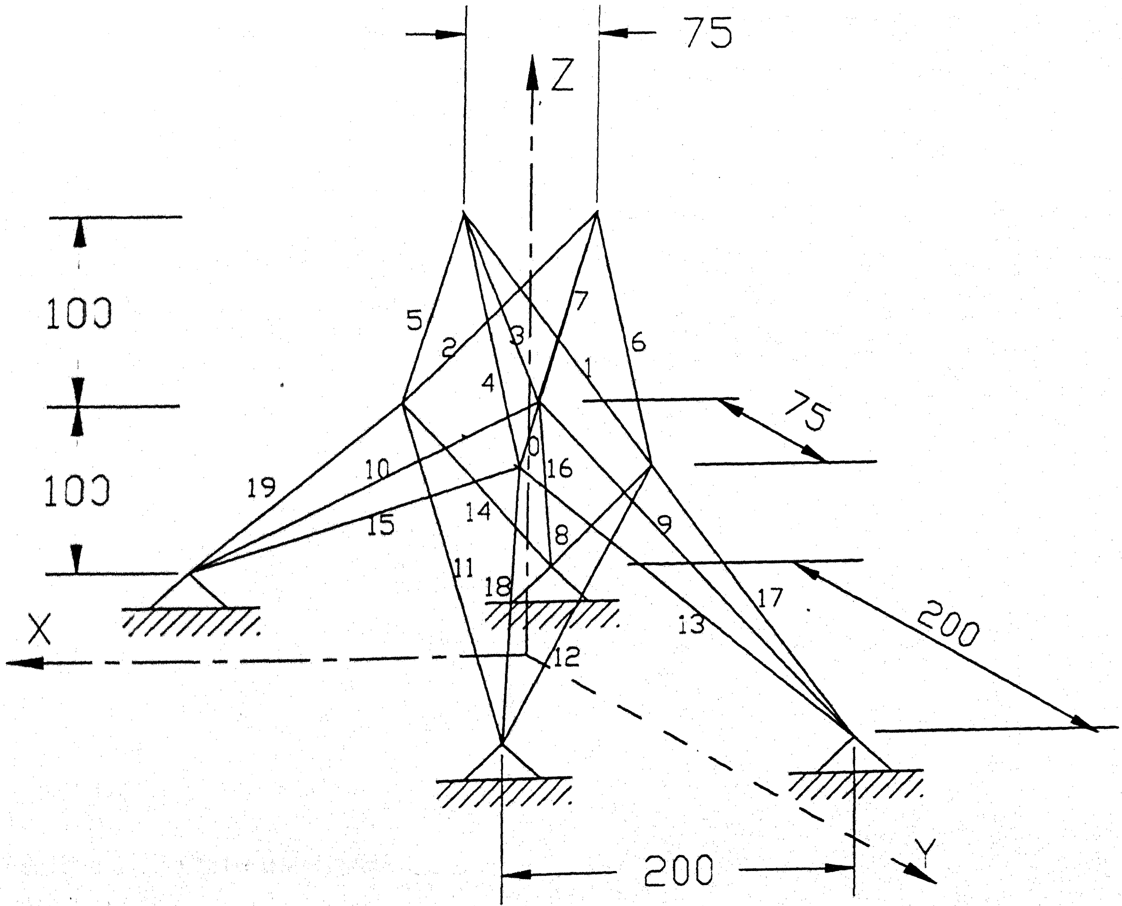


Figure 4.19: Optimized truss for 3D 25-member ground structure using sizing and topology optimization (dimensions are in inches).

Above ground structure is modified after adding 14 extra members, including 8 from top-2 to bottom-4 nodes, and the diagonal members in the middle layer, as shown in Figure 4.20. The additional members are grouped in the following way to increase number of variables by 4.

Group 8: $A_{25}, A_{26}, A_{27}, A_{28}$

Group 9: A_{29}, A_{30}

Group 10: $A_{31}, A_{32}, A_{33}, A_{34}$

Group 11: $A_{35}, A_{36}, A_{37}, A_{38}$

Simultaneous size and topology optimization is carried out for the modified ground structure with 39 members. To investigate the efficiency of proposed algorithm, initially this ground structure is used for simple single loading case with downward loads of 500 lb at both top-most nodes. All parameters are kept as before. Optimized structure obtained in this case is shown in Figure 4.21 and member areas are listed in Table 4.14. In this case the optimized truss does not include the nodes of middle layer, which is intuitive in single downward loading case at both top-most nodes. The GA has been able to remove 30 members and 4 nodes from the ground structure and find an intuitively optimal truss structure.

Member number (refer Figure 4.21)	Areas of member (in^2)
0	0.166
1,2,3,4	0.409
5,6,7,8	0.071
Weight of truss (in lb)	47.93

Table 4.14: Optimized areas for 3D 39-member ground structure in single loading case

The 39-member ground structure (Figure 4.20) is further considered with two loading cases as described before in Table 4.11. Minimum weight of 333.164 lb obtained in this case is much less than that obtained in the 25-member

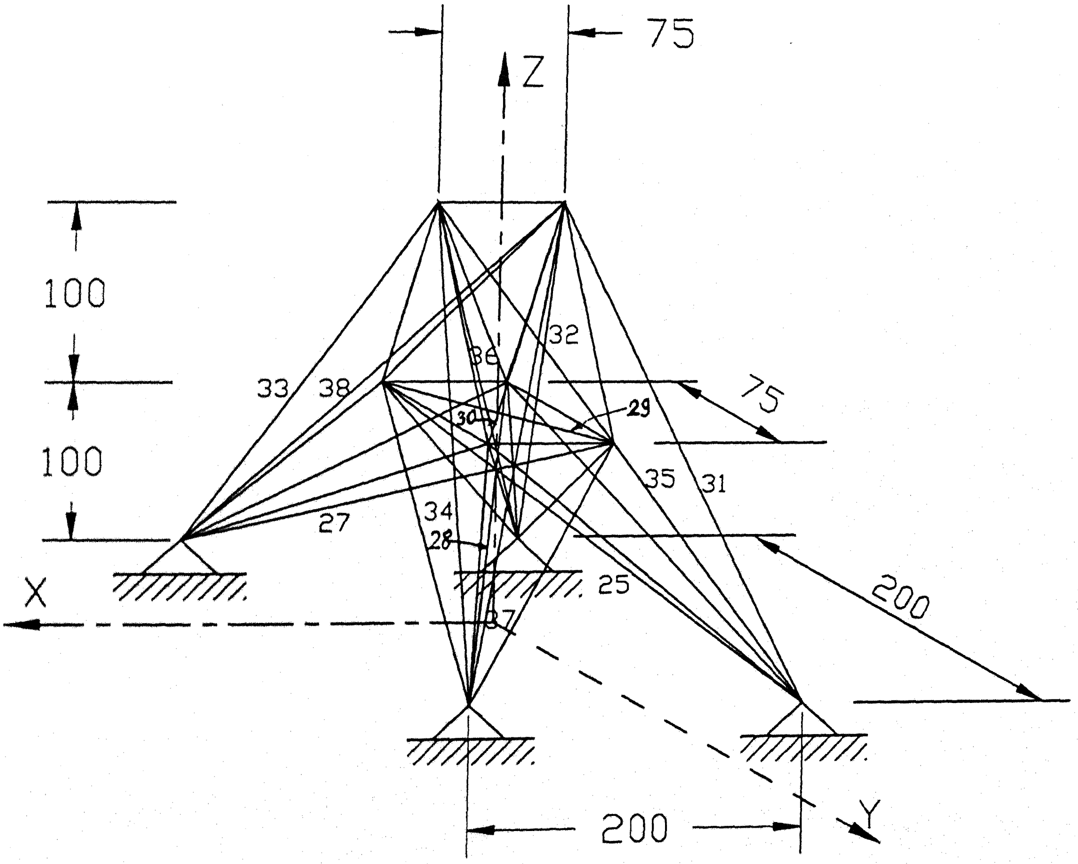


Figure 4.20: 3D 39-member ground structure (dimensions are in inches).

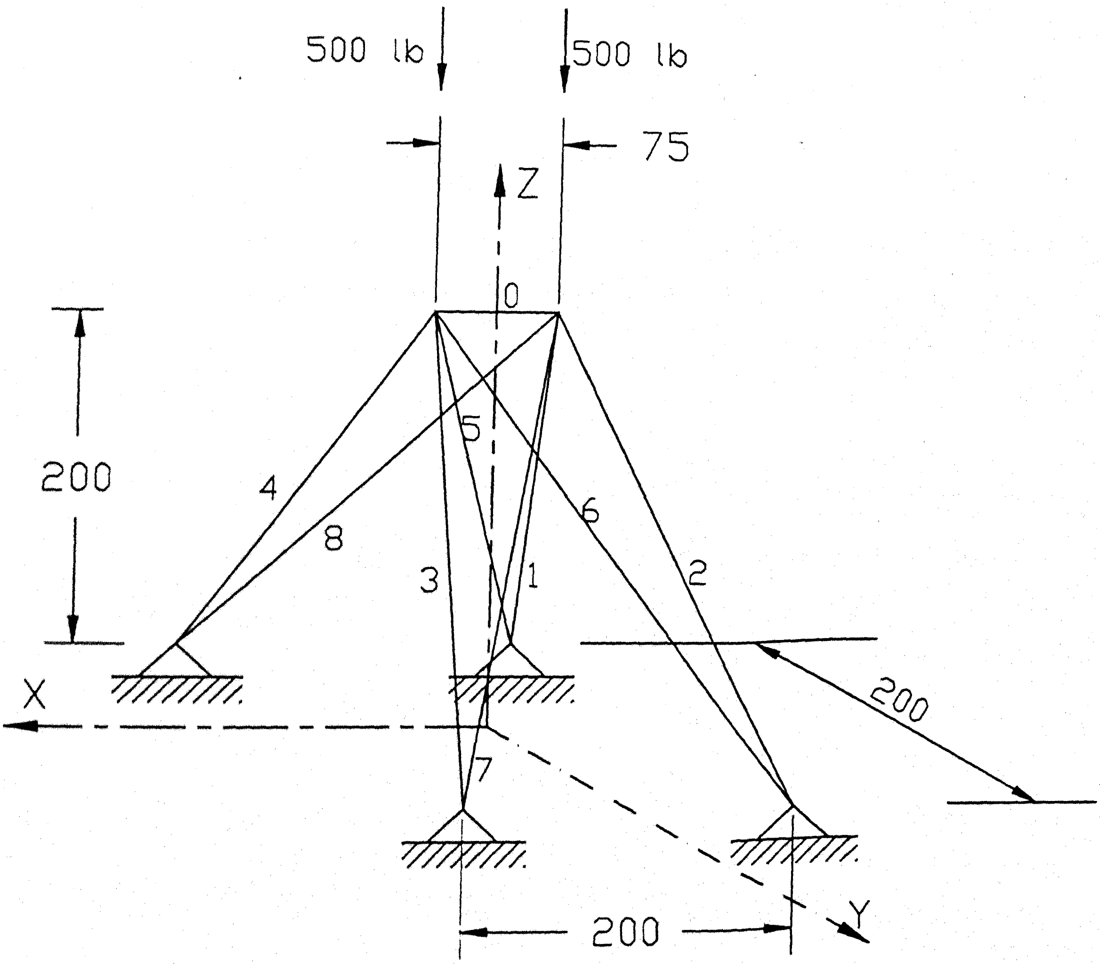


Figure 4.21: Optimized topology for 3D 39-member ground structure for single loading case (dimensions are in inches).

ground structure case (545.167 lb). This is due to the flexibility of choosing extra members in the 39-member case. The optimized structure is shown in Figure 4.22. Optimized member sizing for simulation results with population size of 110 are shown in Table 4.15.

Figure 4.23 shows the generation versus best weight plot for the optimization of 3D 25-member ground structure. This figure shows that GA converges to the final value in 100 generations. Figure 4.24 shows the difference of weight of 39-member ground structure case with those of 25-member cases, using both only sizing, and sizing and topology optimization.

Results in this thesis show that our proposed approach has found better or similar solutions, compared to the existing solutions, in all the 2D and 3D trusses.

Memebr number (refer Figure 4.22)	Areas of member(in^2) population 110
0,1,2,3	0.606
4,5,6,7	0.723
8,9,10,11	0.388
12,13,14,15	2.582
16,17	0.065
18,19,20,21	0.016
Truss Weight (in lb)	333.164

Table 4.15: Optimized areas of truss member for 3D 39-member ground structure.

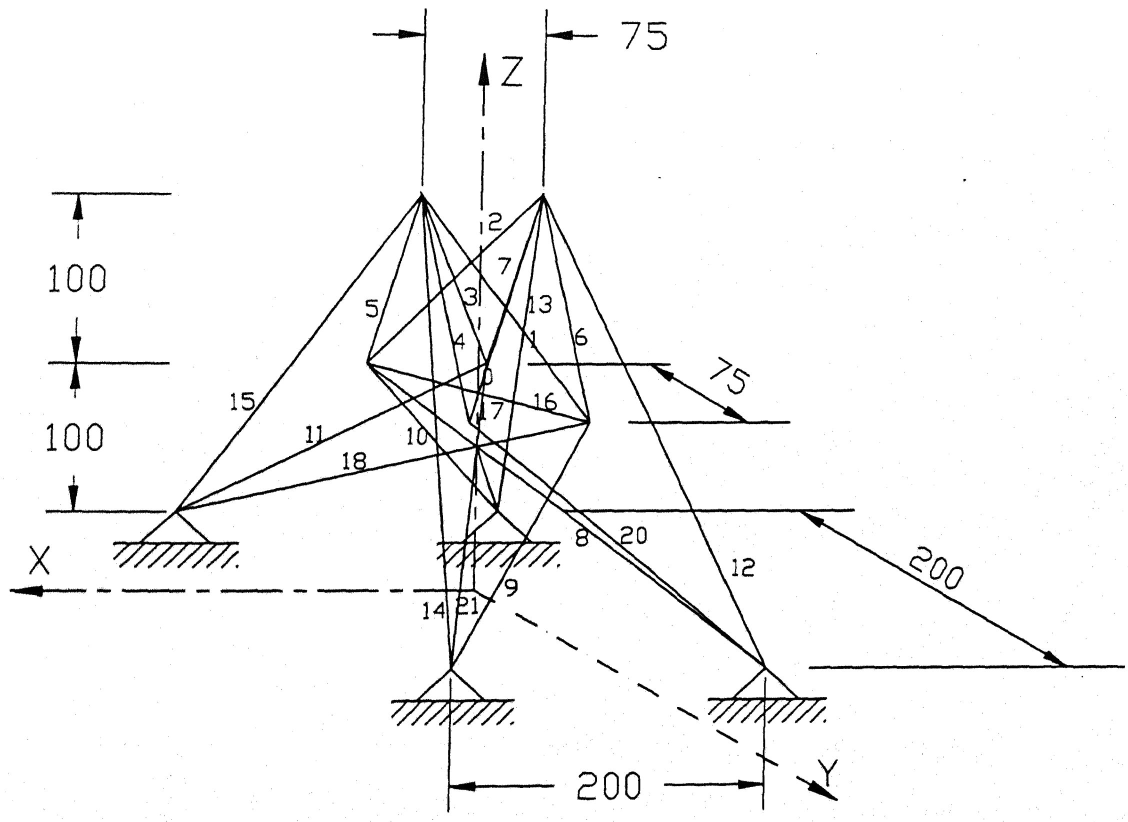


Figure 4.22: Optimized topology for 3D 39-member ground structure (dimensions are in inches).

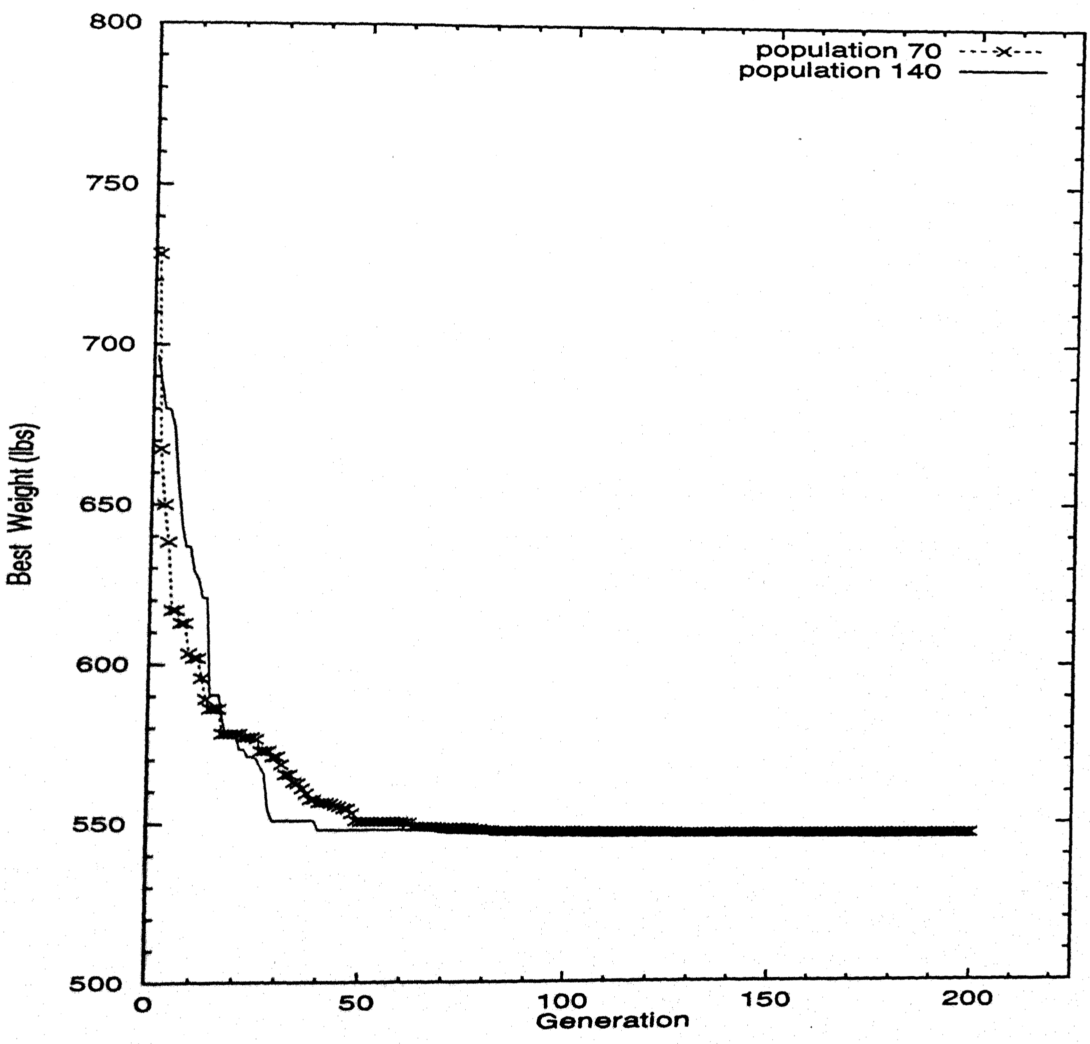


Figure 4.23: Topology optimization of 3D 25-member ground structure.

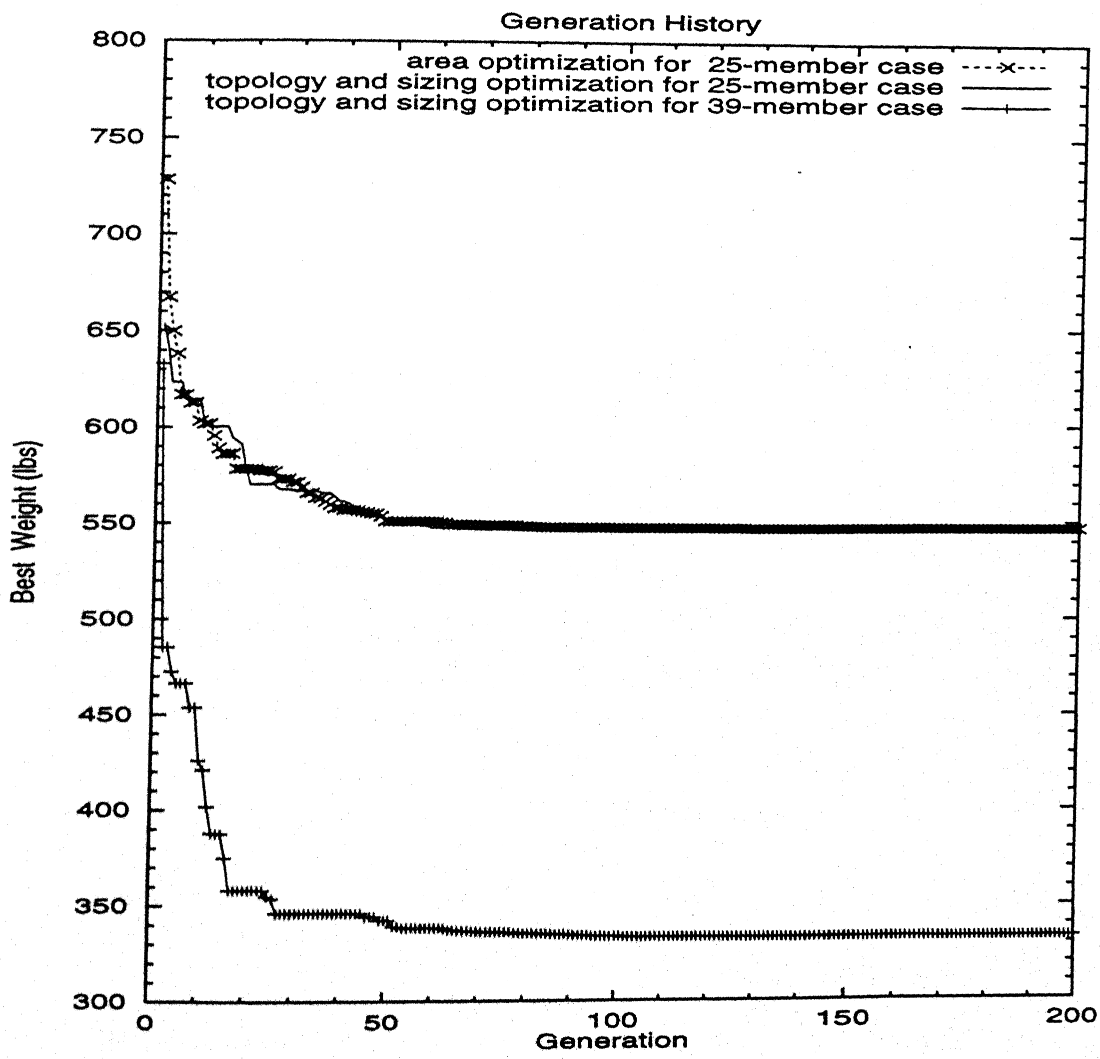


Figure 4.24: Comparison of optimization of 3D 25-member and 39-member ground structure.

Chapter 5

Conclusions

A design methodology is presented for simultaneous optimization of sizing, shape, and topology of trusses. The cross-sectional areas are considered both as continuous and discrete variables for size optimization. Shape optimization is achieved by using nodal coordinates as continuous variables. To obtain topology optimization lower cutoff limit on areas of members is imposed to delete members from the ground structure. Computational expense is minimized by avoiding further calculation of truss in some cases of constraint violation. Comparison of results with that reported in the literature shows that proposed the methodology has been able to find better truss in some problems and match the existing results in other problems.

Based on the result obtained for three classes of optimization problems, the following conclusions are made:

- SBX crossover operator has a good search power since it is able to find optimal or near-optimal solutions despite wide range of design variables.
- Achieving the precision of variables in the solution is easy using real coded GA. As generation proceeds precision increases.
- Population size requirement is dependent on number of variables and the complexity of problem. In the present thesis, population size is taken as $N = cn$ where n is the number of variables, and c is an integer number. Value of c depends upon the complexity of problem. For the ground

structure with greater number of non-basic nodes, complexity of problem is high since more number of stable structures are possible.

- Final optimized results are not much dependent on the chosen initial population. Since GA uses biased stochastic search operator, so effect of initial population diminishes as generation proceeds.

- However, optimized truss structure depends upon the chosen ground structure. If some problem knowledge is used to form the ground structure then the better truss can be obtained.

- Concept of basic nodes plays an important role to reduce the computational efforts and yet create valid trusses having all the nodes having support or loading. This is achieved by returning the penalty and avoiding further calculation in case of any basic nodes is not included.

5.1 Scope of Further Study

The results in the present study using the proposed real-coded GA are encouraging and suggests application of proposed methodology to complex structural optimization problems. Following extensions to present approach are suggested:

- Since structural optimization problems requires large number of variables, these problems requires great computational effort. So to apply the present methodology on large structural problems, parallel implementation of GA is suggested.

- Problem complexity can be reduced by proper ground structure and by using other information such as symmetry and grouping of members, according to loading.

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